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An Aspect of *Katachi* (≃ Form)

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Abstract. Further investigation of geometry is desirable not necessarily as mathematics but as a basis of science. The author's view is set forth. The word *katachi* in the title is a Japanese word that nearly corresponds to *form*, *shape* or *pattern*. For some people, the word *katachi* may sound little intimate with science since the word is categorized rather into those intimate with traditional Japanese culture.

This manuscript is based mainly on two materials. One is the opening lecture at a meeting of this project as an organizer of the subgroup entitled "Nature of Space and its Division". The other is the essay in Japanese by the author concerning with *Symmetry* symposium described in Section 5.

1. Introduction

Watching a drama played by actors and actresses on a stage, a person deduces the minimum script necessary for the drama without mislead by *ad lib* and some impromptu parts. The actors and the actresses are compared to substances or materials, the stage to space, the script to the fundamental natural law and the person to a scientist. The role of a scientist may be caricatured in such a manner. What the author likes to point out about the caricature is about the constraints to the script or the drama which is brought by the facilities of the stage. In other words, the spatial constraints to the phenomena may be rather strong.

Sometimes one says that a phenomenon is understood when the equation describing it is known. Usually, boundary conditions are regarded as less important.

This point of view seems to be accepted as standard. Boundary conditions, however, are sometimes more important, especially in the problems of pattern formation. Taking an example in the pattern formation in Bénard convection phenomena, the importance of spatial constraints is discussed in Section 2. Necessity of more intensive study of Geometry, not necessarily from the stand point of mathematics but as a basis of science, is stressed in Section 3. The question about the time variable is proposed in Section 4. Some comparative discussion is given over *symmetry* in western culture and our *katachi* as a basis of discussions in future in Section 5.

What the author likes to emphasize in this paper is that our knowledge about the nature of the space is too little to be the basis of the science. Though the laws for materials seem to be accepted as more fundamental, it is often difficult to conclude that it is really essential when compared with the laws of space.

2. The Importance of Spatial Constraints

Suppose a shallow and wide vessel is filled with a proper fluid and is slowly heated at the bottom. Heat flows uniformly and temperature raises gradually. In this case, the fluid stays immobile. After a while, if the situation is proper, the fluid starts to move and the mode of heat conduction changes. The wormed fluid flows up, getting light. It is a convection. The flow upward never be uniform since some flow downward of less worm fluid should take place somewhere else. Some cell structure is spontaneously organized. The cells, being units of circulating flows, arrange regularly (Fig. 1). It is called Bénard convection and one of the typical examples of pattern formation (Normand *et al.* 1977).

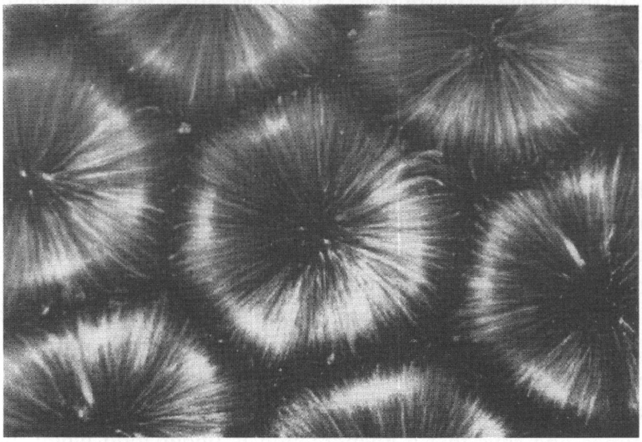


Fig. 1. A regular arrangement in surface-tension-driven Bénard convection cells. [Photograph by M. G. Velarde, M. Yuste, and J. Salan (Velarde, M. G. *et al.*, 1982).

In Bénard convection, for example, the equations which govern the convection are Navier-Stokes equation for mass flow and another equation for energy flow as well as the equation of states expressing such relations between pressures, densities and temperatures as the thermal expansion coefficient and the compressibility. Instability of uniform heat flow toward the convection can be explained by them. But, the shape of the cell, in which mass flow is closed, can not be decided without boundary conditions. Here the most important boundary conditions are not at surroundings but at the upper face.

It is rather difficult to derive theoretically a honeycomb shape in which the number of the cell centers and that of vertices of cells are different; the latter is twice of the former as shown Fig. 2. Therefore, the flow pattern essentially changes if the direction of all flows are reversed. It corresponds to the exchange of two directions, upward and downward. What can bring the asymmetry? As for temperatures and gravity, they both come into the problem only as difference between the values at the upper face and at the bottom. They never bring any asymmetry between upward and downward.

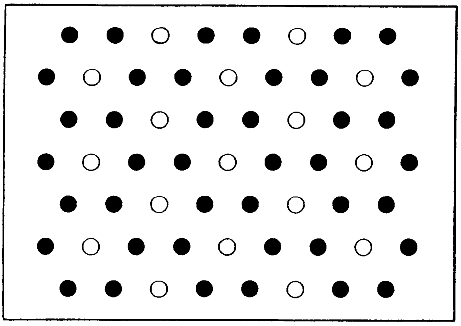


Fig. 2. The centers of upward flow locate at the centers ○ of the hexagonal cells and those of downward flow are at its corners ●. The number of ○ is a half of that of ● in the limit of an infinitely large region.

It is known that the cells of honeycomb shape appear only in open face cases. The upper boundary and the bottom are surely different in open face case. Nevertheless, it is still open question what the open face brings. Surface tension? It is proportional to the area and is considered in the conventional interpretation to be unchanged when the geometrical shape of the face is made upside down.

One may think that it is a kind of broken symmetry occurring spontaneously in a phase transition. Reminisce that the two directions of flow are not equivalent and they are not degenerated. It is the current problem itself.

The purpose of this paper is not a further study of Bénard convection but the raise of a problem concerning the importance of spatial constraints. Regarding the above considerations, which is a kind of *gedankenexperiment*, as an example, the author emphasizes importance of geometrical point of view.

Another example may assist the readers to know the author's view. Plateau problem is one of the most established fields in science of form. It is the problem of minimal surface typically seen in soap films and froths. The fundamental law is as follows. The averaged curvature of a curved face is a constant which depends on the pressure difference between two sides of a face. Three films join one another at exactly 120° to form a line. Four of such lines terminate at a point with the Maraldi's angle of $\cos^{-1}(-1/3)=109°28'16''$. Knowing the fundamental law, one can say a given solution as correct or incorrect. Even if, however, these fundamental laws are known, it is rather difficult to find a solution for a given indivisual condition even in two-dimensional cases (see Fig. 3). In order to find, so to speak, a globally correct solution, the fundamental laws in the conventional sense is not always powerful. Physicists tend to regard an individual problem as included within the general laws. It is desirable to seek the laws of another level in order to fill the gap between various fields of science and culture. The problem is discussed later again in Section 5.

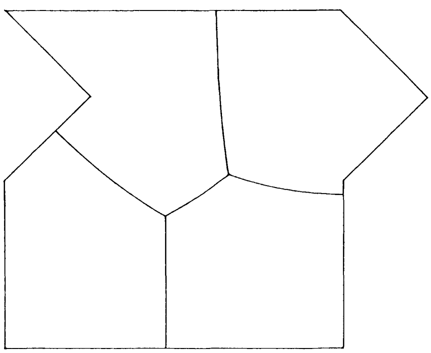


Fig. 3. A problem of puzzle is given "Find the tessellation of a region given in (a) into four parts with the same area so that the total length of the boundaries is the minimum". It is not easy to find the complete answer shown in (b), even if one knows the fundamental laws in conventional sense [I) The shape of a boundary curve is an arc whose radius is proportional to the pressure difference. II) Two boundaries make an angle of 120° at a Y-junction. III) A boundary terminates with right angle at a border of the region.] Another type of laws will be more convenient for practical purposes as some designs.

3. Geometry from the Standpoint of Science

The author is a theoretical physicist with a strong interest in geometrical problems such as structure of liquids, amorphous materials, quasicrystals etc. In

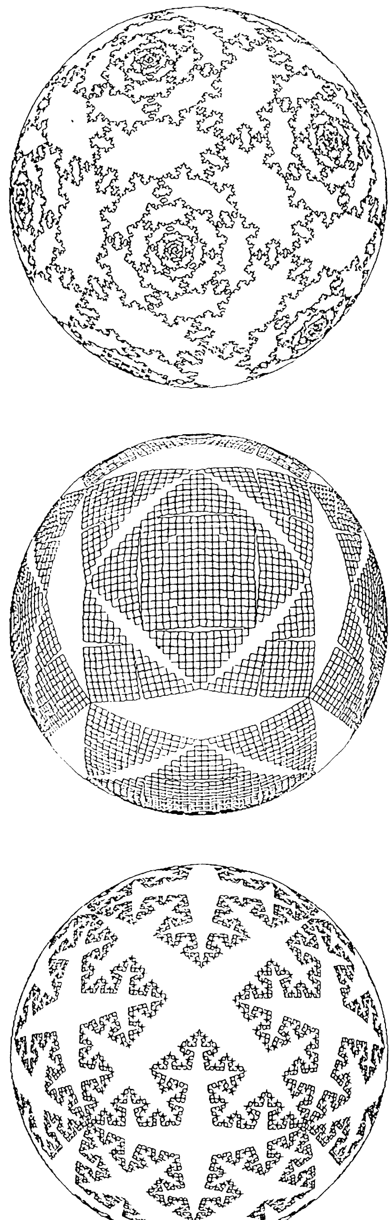
1980, he organized a domestic interdisciplinary symposium "*Morphysics* (originally 形の物理学 *Katachi no Butsurigaku* $\simeq$ *Physics of Form* in Japanese: *katachi* $\simeq$ form, shape or pattern, *no* $\simeq$ of, *butsurigaku* $\simeq$ physics)" in Kyoto together with a physicist Hazime Mori and a mathematician Isao Higuti. Since then, he focuses his research field in geometrical problems. His philosophy is partly published together with a brief review of some pioneering works by early Japanese scientists (Ogawa 1983, 1986a, 1986b, 1987). It is difficult to define *katachi* completely and then an example may be convenient.

Jigsaw puzzle is taken as an example. A jigsaw puzzle is a puzzle of reconstruction of the original or consistent arrangement with the given pieces. A two-dimensional conventional one has a picture or a painting printed on it. Usually, one can get a clue from pictures, without which it is so difficult that only some maniac enjoys it. It is remarkable that the situation is completely different in one-dimension. Suppose a one-dimensional jigsaw puzzle, which is merely a rod or string chopped. If it has not any picture or any marks on it, it is too easy to be a puzzle. Any arrangement, in which direction of a piece and order of pieces are both arbitrary, is a solution. There are $N!2^N$ solutions for N pieces. With a picture on it, it can be a puzzle: It is similar to make a sentence only with the given words. Which is easier or more difficult among two puzzles, the one with picture and the other without picture? The answer to this question is completely different for one- and two-dimension.

The difference comes from something qualitative in two-dimension. In one-dimensional no-picture-case, there must be a solution if the total length of all pieces equals to the length of given *box*. In any two-dimensional case, on contrary, it is no guarantee of existence of a solution that the total area of all pieces equals to the area of given *box* because it is very, very rare that there are two pieces fitting each other. Something qualitative in two- or higher dimensions may be referred to as *katachi*. It is merely an example of *katachi*. It is noted that *katachi* has many other nature.

Though a planar angle is not a simple quantity because of its cyclic properties, the difference is small and the concept of quantity can easily be extended to include planar angle. A solid angle is more close to a two-dimensional jigsaw puzzle and has *katachi*. The difference between planar angle and a solid angle leads to that there are regular n-gons for any integer n and there are only five kinds of regular n-hedra (Platonic solids); only for n=4, 6, 8, 12 and 20. The difference affects the possible arrangements in space and the possible division of space. In order to avoid the confusion between the nature of material and the nature of space, it is necessary to push forward geometry from scientific standpoint. Though a proof is essential from mathematical point of view, any fact is acceptable even if a proof is not attained. It is desirable to get more and more geometrical facts and to systematize them. It is an important branch of science of form.

Before closing this section, two attempts on the fractal tessellation of a spherical surface (Ogawa 1989a, 1989b) are presented in Fig. 4 and 5. It is noted that all the elements filling a spherical surface in Fig. 4 have the *same* shape, where the



a)

b)

c)

Fig. 4. (a) (b) (c) Some examples of fractal tessellation of a spherical surface. Note that the filling elements have the *same shape* (not exactly the same shape because similarity can not be defined in the literal sense for non-eucledian spaces).

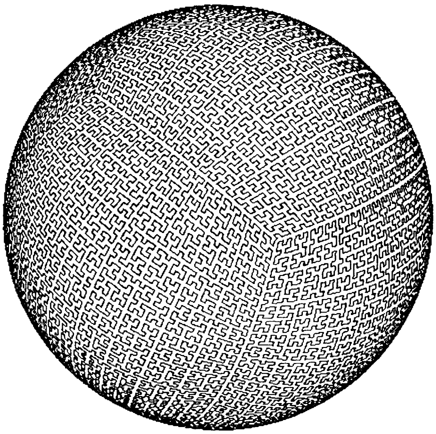


Fig. 5. A Peano curve on a spherical surface. Note that all the points on a spherical surface are expressed by a single parameter.

concept of similarity was extended since there is no concept of similarity in a spherical face being a nonlinear and non-Euclidean space. The attempt was motivated to see how the concept of fractal modifies the problem of division of a spherical surface. Fig. 5 is an extended Peano curve on a spherical surface. See the points on a sphere can be expressed by a single parameter.

4. Static Description also Works for Dynamics

The philosophy of the author is sometimes misunderstood. People seem to think that the author's concept of *katachi* is static and the dynamical problems are out of his scope.

Dynamical problems are also interesting to the author. He, however, thinks that it is not necessary to treat time variable differently from other spatial coordinates. Regarding time as a parameter is only a single picture among many many ways. A motion is often described in a space, which contains time as an axis. In this case, time is treated as a spatial variable. In some problems, everything gets clear by knowing the structure of phase space, which corresponds to know the connectivity between all states. It is not necessary to see things dynamically in treating dynamical phenomena. What is important in treating dynamics is not to give a special role to time but to know the connectivity or the relations in high dimensions correctly. What is essential in *katachi* is not necessarily space in a narrow sense but how to treat or how to recognize the connectivity or the relations in high dimensions correctly. *Katachi* is connection as a melody is so instead of merely a set of sound elements.

In a symposium of this project, Isao Higuti put the following comment. Even in differential calculus which was developed together with dynamics, the definition of differential coefficient is rather static in the ε-δ logic which is based not on change itself but on the existence of δ for any ε.

So long as the dynamical problems in usually treated concern, the above mentioned picture is enough. A genuine problem lies in whether there is any essential difference between spatial variables and time. Of course, if the sight is restricted only within isotropic spaces, the picture is too narrow to treat to dynamics.

5. Symmetry and Katachi

An international interdisciplinary symposium "Symmetry of Structure" was held in Budapest in august of 1989 (Darvas and Nagy 1991). Shozo Ishizaka and I were invited to the symposium by the organizer perhaps as the organizers of the symposium "Science on Form", which, being also international and interdisciplinary, was held in Tsukuba in November of 1985 (Toda 1986, Ogawa 1986a). The underlying philosophies of these two symposia seem to be basically the same or to have many common aspects. The following are my personal impression, feeling, philosophy and/or ideas. I keenly wish if I had enough power in my expression in English or an international language. Anyway, I am trying to write frankly my thinking in order to propose a basis for discussions in future.

No doubt, I felt sympathy and even a kind of resonance in early 1988 at reading the prospectus of the symposium in Budapest. We should construct a bridge over the crack opened between science and art as Dénes Nagy, who is one of the organizers and a Hungarian scientist working in various fields, for example mathematics and history of science. [Here, *science and art* may be replaced with *two cultures* by C. P. Snow (Snow, 1964).] For Nagy, *symmetry* is the key concept for this purpose. Nevertheless, I had been hesitating for a year and half until I finally decided my mind to attend the symposium immediately before it. The origin of my hesitation surely lies in the following idea.

I myself had been feeling a similar necessity of making a bridge over the crack opened between science and art. Since our organization of domestic interdisciplinary meeting *Morphysics* in 1980, I have been thinking that the concept of *katachi* may play an important role for this purpose. The origin of the success of physical science lies probably in its methodological nature to be quantitative as sometimes described. It is necessary to extend the framework of physical science to cover the problems in which quantification is not so easy (Ogawa 1986b). Of course, there are many fields in science in which quantitative method is not proper. Even in such fields, however, the existence of the tendency to get more quantitative can not be denied. It seems to me very important to try many *avantgarde* attempts in science to find a breakthrough toward some soft fundamental science (Ogawa and Nakajima 1986; Ogawa *et al.* 1991). Some metaphorical models, for example, should be more widely accepted in physical science in order to avoid the cracks, which are distributed over the culture. Can the concept of *katachi* play an important role for this purpose? It is surely a big question to me. Anyway, I am trying *science of katachi* now.

Though *symmetry* has surely some close connection with *katachi*, it sounds, more or less, restrictive at least for those people with the background of Japanese culture as a matter of fact. I feel that *symmetry* is not the required thing itself. Some fields can not be a science without the concept of *symmetry* and/or group theory, for example, crystallography, nuclear physics and particle physics. However, the *symmetry* in such a restricted sense is very difficult to go further because it has almost completed already. *Symmetry* sounds to me as such a concept.

I can not say now explicitly what is *science of katachi*. I can not say explicitly how can we attain science of *katachi* now. *Culture of Katachi* may be far more attainable in some sense. If even so, I, however, can not say now explicitly how can *culture of Katachi* be *science of Katachi*.

It seems not so curious for Japanese people to talk over *katachi* as a complimentary concept to science, especially when one thinks a scientific way of thinking is not always so perfect. In other words, I dare to say that many Japanese scientists often feel that a scientific paper is not so complete in the sense that it does not cover everything that they think important. In a sense, *katachi* may be a symbol of something to supply the western logic: being quantitative too much and being two-valued to too much extent (always either of yes or no). In Japanese language, the word *symmetry* is usually translated as 対称性 (*taishôsei*). According to my own linguistic feeling, this word sounds as a cast one for the necessity in translation. Of course, in a practical sense, symmetry has traditionally been used everywhere also in Japan. But, in conceptual sense, *symmetry* has not been thought as so important. *Harmony* and *balance* have been more explicitly accepted as important.

By the way, the following quotation (Feyman *et al.* 1963) suggests the position of *symmetry* in Japanese culture.

> "So our problem is to explain where symmetry comes from. Why is nature so nearly symmetrical? No one has any idea why. The only thing we might suggest is something like this: There is a gate in Japan, a gate in Neiko, which is sometimes called by the Japanese the most beautiful gate in all Japan; it was built in a time when there was great influence from Chinese art. This gate is very elaborate, with lots of gables and beautiful carving and lots of columns and dragon heads and princes carved into pillars, and so on. But when one looks closely he sees that in the elaborate and complex design along one of the pillars, one of the small design elements is carved upside down; otherwise the thing is completely symmetrical. If one asks why this is, the story is that it was carved upside down so that the gods will not be jealous of the perfection of man. So they purposely put an error in there, so that the gods would not be jealous and get angry with human beings."
>
> [The gate is *Yômeimon* in *Nikkô* (not *Neiko*) which was built in 17th century.]

Attending the symposium in Budapest, I had the following impression about *symmetry*. *Symmetry* lies far more deeply in the western culture than I had been thinking so far. If based on western culture: Anything can not be systematic without the notion of *symmetry*; To find a natural law is to find a symmetry; Anything recognizable may be inevitably something symmetrical; Symmetry may be everything; *Symmetry* may be science or culture itself.

If really so, extremely to say, it is nearly equivalent to express nothing. The organizers of the *symmetry* symposium, of course, stressed the necessity of generalizing the notion of *symmetry* to include more soft or fuzzy things. I felt it very very natural that they take the starting point at *symmetry* since they can not think anything in any other way. Taking a view of the garden in Belvedere Palace in Wien (Fig. 6), I dreamed that the owner might have been irritated at the arrangement of the towers of chapels outside the palace which breaks too much perfect symmetry inside the palace.

According to the above idea, any *culture of Katachi* may not be able to be *science of Katachi*, without *symmetry*. Is it really true? Trying both ways may be necessary. Of course, cooperating with each other, covering each other and accepting another standpoint with each other, to take other way to each other, is really important. It is often said that science is objective and universal. I agree with the statement. It is, however, true that there are some facts which are quite easily

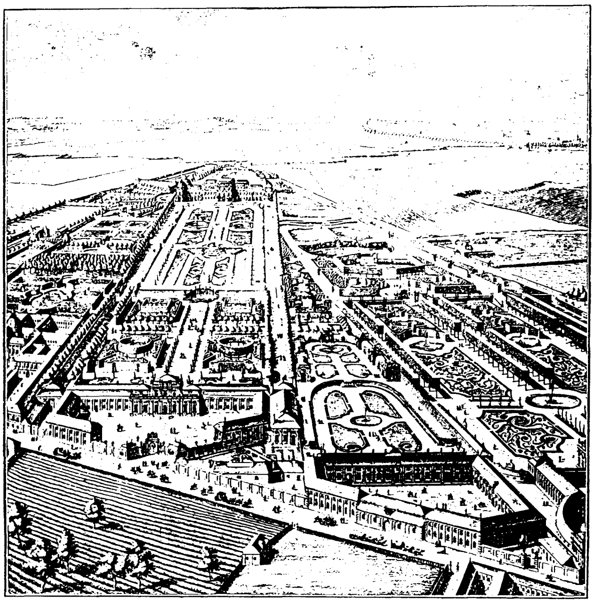
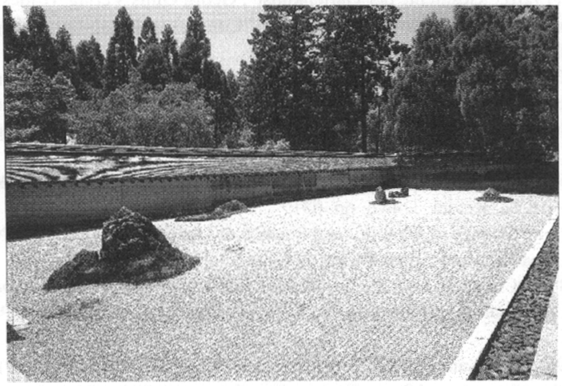


Fig. 6. A view of the garden in Belvedele Palace in Wien. (Gotheim, 1926).

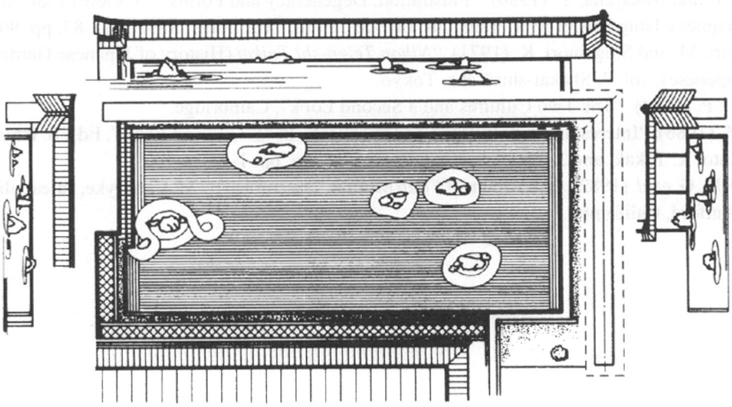
seen on one cultural background but difficult on other cultural background. After once recognized, they can be translated into other cultural background and is acceptable to every one. I personally regard the objectivity and universality of science as such kind of matter.

It is my opinion about symmetry that the most important is not symmetry itself but between what and what one finds something essential and common to different things. From what point of view, is it possible?

Such a concept that can scientifically treat the order lying in a typical Japanese garden (Fig. 7), is now required even in physics.



(a)



(b)

Fig. 7. A stone garden in Ryôanji temple in Kyoto (A typical Japanese garden) (a) The photograph, which was taken in May, 1986, is used by the courtesy of Ryôanji-temple in Kyoto. (b) The plan was roughly traced from a measured map made in 1938 (Shigemori and Shigemori, 1971).

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