*Research of Pattern Formation*, edited by R. Takaki, pp. 343–361.


# Pattern Formations in Magnetically and Electrically Induced Freedericksz Transitions and in Multiplicative Stochastic Processes

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**Abstract.** The steady and transient pattern formations are described theoretically and experimentally in the Freedericksz instabilities magnetically and electrically induced in nematic liquid crystals. The transient pattern formations can be explained by the backflow effect due to the director reorientation when considerably large external fields are applied. Under the situation superimposing fluctuating fields (external noise) with deterministic fields, called multiplicative noise effects, the response of the instabilities is changed. The threshold field for the onset of the instabilities decreases with increase of noise fields, as expected previously. The solitons and wall motions are also described briefly when a rotating magnetic field is applied to the homeotropic orientation sample. Most of them are rather new phenomena and not well understood yet.

## 1. Introduction

Pattern formations in dissipative systems are very interesting subjects to be investigated. Especially they are usually dynamic and show various beautiful patterns. One example of them which has been well studied is the electrohydrodynamic instability (EHDI) in anisotropic fluids, nematic liquid crystals (de Gennes, 1982; Blinov, 1983; Kai and Hirakawa, 1978). In EHDI however, pattern formations are much complicated to be understood because of anisotropies and existense of convections. In spite of these disadvantages as they show very similar features to those in the well-known Rayleigh-Bénard convection, qualitative understandings are now in progress quickly and they have been extensively studied by many authors recently (Kai and Zimmermann, 1989; Kramer *et al.*, 1989; Joet and Ribbota, 1986; Rehberg *et al.*, 1988).

The rich behavior of nematic liquid crystals with respect to their pattern forming instabilities can be better understood by presenting an example quite apart from the context of EHDI. In the particular situation, we are going to deal, theoretically with on what follows, the external forcing agent is a magnetic field which causes an internal reorientation of the sample, which is again coupled to flow motions inside the material. The same situation happens when the magnetic field is just replaced by an electric field as will be experimentally described here. These reorientation instability of director is called the Freedericksz transition.

In ideal situations, both electrically and magnetically induced Freedericksz transitions are identical with each other for theoretical consideration. Here we describe their dynamics in both fields. Especially in an electric field, the study of external noise effects called multiplicative stochastic processes is extremely convenient. Therefore, in this article we will also describe multiplicative stochastic processes in the electrically induced Freedericksz transition as well as in EHDI.

## 2. The Freedericksz Transitions

### *2.1 Transient pattern formations in Freedericksz transition*

The magnetically induced Freedericksz transition occurs in a nematic slab when the director reorientates with an angle $\theta$ in the direction of an applied magnetic field $H$ larger than a critical one $H_c$ (Fig. 1). The standard description corresponds to the appearance of distortions in the orientation of the nematic molecules with respect to the original configuration, the degree of distortion being homogeneous

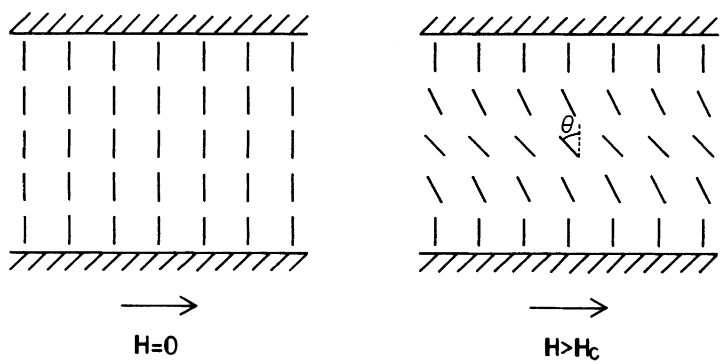


Fig. 1. The magnetically induced Freedericksz transition.

in each plane of the sample, and of a maximum intensity in the mid plane far from the plates limiting the nematic materials.

This simple picture of the Freedericksz instability is however too simplified to account for some experimentally observed facts. A particularly interesting feature in this context concerns the occurrence of transient spatial structures corresponding to modes of inhomogeneous distortions on the planes of the sample. There is now a broad experimental evidence of this phenomenon (Guyon *et al.*, 1979; Lonberg *et al.*, 1984; Hui *et al.*, 1985; Hurd *et al.*, 1985; Kuzma, 1986; Wu *et al.*, 1990), which also deserved a detailed theoretical analysis (Guyon *et al.*, 1979; San Miguel and Sagues, 1987, Sagues, 1988, Sagues *et al.*, 1988). The suggested explanation involves a dynamical coupling between the director field and the hydrodynamic motion associated with the reorientation. Such a coupling gives rise, during the transient process, to spatial domains with a well-defined periodicity. In these domains the director field reorientates in opposite but equivalent directions. The selection of a wavenumber is thus associated with the dynamics of a symmetry breaking. This phenomenon has been observed for different materials, both thermotropics and lyotropics, as well as for different geometries regarding the initial orientational configuration of the sample and the applied magnetic field (Guyon *et al.*, 1979; Lonberg *et al.*, 1984; Hui *et al.*, 1985; Hurd *et al.*, 1985; Kuzma, 1986; San Miguel and Sagues, 1987, 1988a, b; Srajer *et al.*, 1989; San miguel and Sagues, 1990). In the simplest cases the pattern consists in a collection of stripes perpendicular to the initial director (Guyon *et al.*, 1979; Lonberg *et al.*, 1984; Hui *et al.*, 1985), although oblique (Hurd *et al.*, 1985) and two-dimensional structures have also been detected (Kuzma, 1986).

The characteristic periodicity of these transient patterns has been commonly described in terms of a most unstable mode (Guyon *et al.*, 1979; Lonberg *et al.*, 1984; Hui *et al.*, 1985; Hurd *et al.*, 1985; Kuzma, 1986). A linear analysis of the nematodynamic equations around the initial undistorted configuration identifies the mode of fastest growth. It is assumed that this mode dominates the transient dynamics. Its characteristic wavelength is associated with the observed periodicity. The dependence of this wavelength with respect to the applied magnetic field seems to be in agreement with experimental observations. However, this approach although useful understanding the main physical ingredients in the origin of the observed periodic structures, is far less valid if one is interested in the dynamics of the pattern formation process itself. For this reason, we have recently made the nonlinear nematodynamic model in the reorientation of the nematic sample. This model enables us to study the time dependence of the characteristic periodicity starting from the homogeneous sample as initial configuration. The subsequent analysis is based on the evolution equation for the time-dependent structure factor which accounts for the orientational distortions of the director, once thermal fluctuations and hydrodynamic effects have been taken into consideration.

Thermal fluctuations, essential in triggering the initial decay from an unstable state, are incorporated in our description through the use of Langevin-type equations corresponding to a time-dependent Ginzburg-Landau (TDGL) formulation commonly invoked in studying critical dynamics and the dynamics of phase transitions (Gunton *et al.*, 1983). In this way our approach to the problem of the Freedericksz transition is reminiscent of the Cahn-Hilliard-Cook theory of spinodal decomposition (Hohenberg and Nelson, 1979). However, an important remark is that, taking the mean first passage time as an indicator, the equations we proposed for the Freedericksz problem admit linearization procedures that turn out to be valid not for the whole process but for considerably larger time scales as compared to the case of spinodal decomposition for systems with short range forces. This should make our predictions much more easily accessible to experimental testing.

On the other hand, it is worth remembering that, in the spinodal decomposition problem, the fact that the most unstable mode is not the homogeneous one can be understood in terms of a conservation law. In the Freedericksz transition, however, this effect can be related to a tradeoff of rotational for shear viscosities, leading to a compromise at some intermediate nonzero wavenumber, for which the increase in elastic energy contribution is favorably balanced by a higher energy dissipation rate, controlled by a lower effective viscosity. This effect, as mentioned above, directly results from the coupling of the director rotation and fluid velocity gradients. In what follows, and from a theoretical point of view, we will describe this behavior as one of the simplest realizations of the magnetically induced Freedericksz instability.

Let us consider the twist geometry described in Fig. 2. The sample is contained between two plates perpendicular to the $z$ axis. The director is initially aligned along the $x$ axis, and the applied magnetic field is aligned along the $y$ axis. We want to study the transient behavior of the system when the magnetic field is switched at $t = 0$ from an initial value below $H_c$ to a final value above it. The physical picture of the formation of a transient pattern is also indicated in this figure. For positive

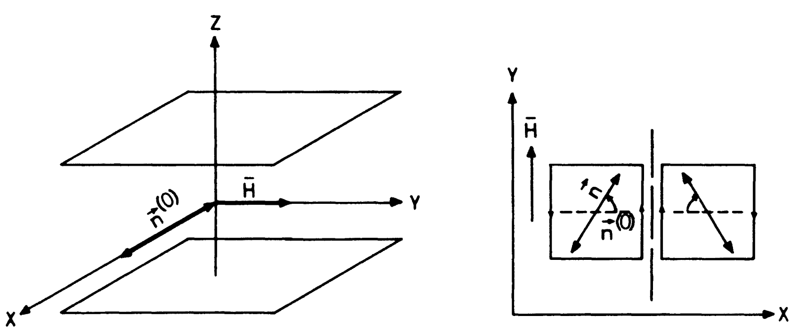


Fig. 2. Schematic representation of the geometry of the nematic sample. Flows generated by oppositely rotating zones which explain the appearence of transient structures are also schematically displayed.

diamagnetic anisotropies, the director tends to become parallel to the magnetic field. For high enough fields this reorientation may occur locally in opposite but equivalent directions. We assume, for simplicity, that macroscopic flow only exists in the $y$ direction, and homogeneity extends on the direction of the applied magnetic field. We finally assume that the director reorientates on the $x$-$y$ plane:

$$n_x(x,z) = \cos\phi(x,z), \tag{1a}$$

$$n_y(x,z) = \sin\phi(x,z), \tag{1b}$$

$$n_z = 0. \tag{1c}$$

A minimal coupling approximation is then invoked (Sagues *et al.*, 1988; Sagues and Arias, 1988; San Miguel and Sagues, 1990) to convert the general nematodynamic equations into a pair of closed equations for $\phi$ and $v_y$:

$$d_{t'}\phi = (-1/\gamma_1)\delta F/\delta\phi + (2\rho)^{-1}(1+\lambda)\partial_x \delta F/\delta v_y + \xi(\bar{r},t'), \tag{2a}$$

$$d_{t'}v_y = (2\rho)^{-1}(1+\lambda)\partial_x \delta F/\delta\phi + \rho^{-2}\left(\nu_2\partial_z^2 + \nu_3\partial_x^2\right)\delta F/\delta v_y + \left(\partial_x\Omega_{yx} + \partial_z\Omega_{yz}\right) \tag{2b}$$

$$\delta F/\delta\phi = \left[K_{22}\partial_z^2\phi + K_{33}\partial_x^2\phi + \chi_a H^2\left(\phi - (1/3)\phi^3\right)\right], \tag{2c}$$

$$\delta F/\delta v_y = \rho v_y. \tag{2d}$$

The Gausiann random forces appearing in the above Langevin-type equations satisfy fluctuation-dissipation relations in terms of pure rotational and shear viscousities, respectively $\gamma_1$ and $\nu_2$, $\nu_3$. $L$ and $\rho$ stand respectively for a linear transversal dimension and the mass density of the sample:

$$\left\langle \xi(\bar{r},t_1')\xi(\bar{r}',t_2')\right\rangle = (2k_BT/\gamma_1 L)\delta(x-x')\delta(z-z')\delta(t_1'-t_2'), \tag{3a}$$

$$\left\langle \Omega_{y\alpha}(\bar{r},t_1')\Omega_{y\beta}(\bar{r}',t_2')\right\rangle = \left(2k_BT/\rho^2 L\right)\nu_\alpha\delta_{\alpha\beta}\delta(x-x')\delta(z-z')\delta(t_1'-t_2'). \tag{3b}$$

$$\left(\nu_{x,z} = \nu_{3,2}, \quad \alpha,\beta = x,z\right)$$

A series of technical manipulations are then strictly convenient in order to proceed further with the analysis of the early stage dynamics of the Freedericksz transition. First of all one introduces the hypotheses of negligible inertia, which enables us to obtain a closed equation for the deformation angle. This equation is more easily handled in terms of a Fourier representation appropriate to the strong anchoring boundary conditions prescribed for the nematic molecules at the limiting plates, $z = \pm d/2$:

$$\phi(x,z;t') = \sum_{m}\sum_{q_x} \theta_{m,qx}(t')\cos(2m+1)\pi z / d \cdot \exp(iq_x x), \tag{4a}$$

$$\xi(x,z;t') = \sum_{m}\sum_{q_x} \xi_{m,qx}(t')\cos(2m+1)\pi z / d \cdot \exp(iq_x x), \tag{4b}$$

$$\Omega_{y\alpha}(x,z;t') = \sum_{m}\sum_{q_x} \Omega^{\alpha}_{m,qx}(t')\cos(2m+1)\pi z / d \cdot \exp(iq_x x). \tag{4c}$$

$$(\alpha = x, z)$$

The resulting equation for the amplitude of the reorientational mode even in its linear version, already shows the dynamical consequences of the reorientation-flow coupling

$$\partial_{t'}\theta_{m,qx}(t')$$

$$= \gamma_{1q}^{-1}\left[\chi_a H^2 - K_{22}(2m+1)^2(\pi/d)^2 - K_{33}q_x^2\right]\theta_{m,qx}(t') + \eta_{m,qx}(t'), \tag{5a}$$

$$\gamma_{1q} = \gamma_1 - \alpha_2^2 / \left(\eta_c + \eta_a q_T^{-2}\right), \tag{5b}$$

$$\eta_{m,qx}(t')$$

$$= \xi_{m,qx}(t') + \left(\alpha_2\rho / \left[\gamma_1\left(\eta_c + \eta_a q_T^{-1}\right) - \alpha_2^2\right]\right)\left[\Omega^{x}_{m,qx}(t') - iq_T^{-2}\Omega^{z}_{m,qx}(t')\right], \tag{5c}$$

$$q_T = q_x d / \pi(2m+1), \;\; \alpha_2 = -\gamma_1(1+\lambda)/2, \;\; \eta_a = \nu_2, \;\; \eta_c = \nu_3 + \gamma_1(1+\lambda)^2/4.$$

The important point to be noticed is that the temporal evolution of the reorientational process is no longer dictated by the pure rotational viscosity $\gamma_1$ but is governed by an effective wavenumber dependent viscosity $\gamma_{1q}$. Thus, the coupling of the director and velocity fields results in a reduction of the viscosity for all modes $q_x \neq 0$. This permits modes of bend deformation along the $x$ direction to grow faster than the homogeneous one, giving rise to pattern structuration. Actually, using literature values for a typical nematic material, MBBA at room temperatures ($\alpha_2^2/\gamma_1\eta_c = 0.74$, $\eta_a/\eta_c = 0.40$) it is easy to see that $\gamma_{1q} = \gamma_1$ when $q_x = 0$, and a minimum $\gamma_{1q} = 0.26\gamma_1$ for $q_x \to \infty$. ($\gamma_{1q}$ is always positive with a maximum value.) Thus, the stability range for the different modes ($q_x$, $m$) remains unmodified with respect to the case without hydrodynamical coupling: For $H > H_c = (K_{22}\pi^2/(\chi_a d^2))^{1/2}$, twist $m$ modes become unstable. However, due to the dependence of $\gamma_{1q}$ on $q_x$ modes of fastest response $q_x \neq 0$ may lead the response of the system provided,

$$h^2(m) = H^2 / \left[(2m+1)^2 H_c^2\right], \tag{6a}$$

$$h^2(m) > 1 + \left(K_{33}\gamma_1\eta_a / K_{22}\alpha_2^2\right). \tag{6b}$$

Accepting that the mode of fastest response slaves other modes during the transient evolution following the reorientation, one predicts the appearance of a periodic bend pattern for magnetic fields satisfying the above condition. This occurs for fields not much larger than the critical one $h^2(0) = 1$ (normalized field), although there still exists a range of fields for which the homogeneous response dominates.

The early dynamical stages of the transition can be easily followed by converting Eq. (5) above into an equation for the structure factor $C_{qx,m}(t') = \langle\theta_{qx,m}(t')\theta_{-qx,m}(t')\rangle$. Using standard methods one has,

$$\partial_{t'} C_{qx,m}$$

$$= \left(2C_{qx,m}(t') / \gamma_{1q}\right)\left[\chi_a H^2 - K_{22}\left((2m+1)\pi / d\right)^2 - K_{33}q_x^2\right] + 4k_B T / \gamma_{1q} V. \tag{7}$$

A convenient way of monitoring the dynamical emergence of the pattern consists in analyzing the time evolution of the mode $q_{T\max}$ corresponding to the maximum of the structure factor $C_{qx,0}$ ($m = 0$ is the most unstable twist deformation mode). This is depicted in Fig. 3. Different and well-resolved time scales can be distinguished in this figure. A first well-defined time scale corresponds to the sharp increase of $q_{T\max}$, when the system takes off from the initial conditions. This time is associated with the characteristic time at which the periodic pattern appears. A second time scale can be identified as corresponding to the slow growth of $q_{T\max}$, which is reasonably associated with the formation and development of the spatial pattern. Late stage dynamical scales for the disappearance of such transient patterns

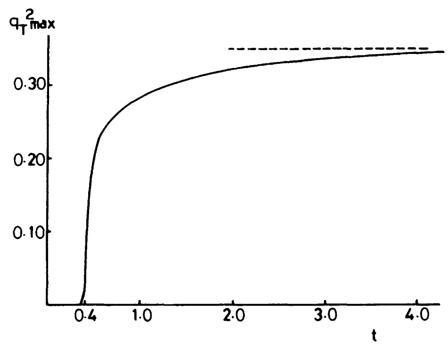


Fig. 3. Temporal development of wavenumber corresponding to the maximum of the structure factor (see text).

cannot be described within the limits of this approach and mobility and recombination of defect walls must be taken into account (Sagues and San Miguel, 1989).

Different situations with interesting dynamical implications can be envisaged when considering the Freedericksz transition under non-standard magnetic forcings. In particular, we will refer on what follows to a pair of particular examples concerning respectively a fluctuating and rotating magnetic field.

*2.2 The Freedericksz transition under a rotating magnetic field*

Very interesting theoretical and experimental implications can be envisaged when one refers to the magnetically induced Freedericksz transition conducted under a rotating magnetic field. For an homeotropic geometry, which is the one discussed here, experiments were already performed by Brochard *et al.* (1975), whereas a theoretical discussion can be found in Brochard *et al.* (1975) and Sagues (1988).

As before we take $z$ as the direction perpendicular to the plates containing the sample. According to the prescribed homeotropic alignment of the directors the sample is subjected to a rotating magnetic field applied perpendicularly, let us say in the $x$-$y$ plane. If the intensity of the magnetic forcing exceeds an $\omega$-dependent threshold, to be later determined, the sample will undergo an internal homeotropic-planar reorientation. The dynamical equations appropriate to the situation here considered can be better formulated in terms of two polar angles, the distortion and azimuthal angles $\theta(r, t')$ and $\phi(r, t')$ respectively. A convenient reduction in the level of technical complexity of the equations to be used can be attained under the reasonable assumption which amounts to neglecting all spatial inhomogeneties for the azimuthal variable. Proceeding in this way the equation for $\phi(t')$ decouples from the one for the distortion angle $\theta(r, t')$, and enables us to readily identify two different regimes regarding the rotation of the director:

1) A synchronous rotation characterized by a constant retardation angle $\alpha$:

$$\phi(t) = \Omega t - \alpha, \quad t = \tau_0^{-1} t', \tag{8a}$$

$$\sin 2\alpha = \Omega\tau, \tag{8b}$$

$$\tau = 2 / h^2, \quad \tau_0 = \frac{\gamma_1}{\chi_a H_c^2} = \gamma_1 d^2 K_{33} \pi^2, \tag{8c}$$

$$\Omega = \omega\gamma_1 d^2 / K_{33}\pi^2. \tag{8d}$$

2) An asynchronous rotation characterized by a time-dependent retardation angle $\alpha(t)$:

$$\phi(t) = \Omega t - \alpha(t), \tag{9a}$$

$$\tan\alpha(t) = (\Omega\tau)^{-1} + \left[1 - \Omega^2\tau^2\right]^{1/2} \tan\left[\left(\Omega^2\tau^2 - 1\right)^{1/2} t / \tau\right]. \tag{9b}$$

On what respects to the onset of the Freedericksz instability one finds two well-different behavior (Brochard *et al.*, 1975; Sagues, 1988):

In the synchronous case the critical Freedericksz field is a nonlinear function of the rotation frequency of the applied magnetic field:

$$H_c(\omega)^2 / H_c^2 = 1 + \left(\gamma_1 \omega d^2 / \pi^2 K_{33}\right)^2 \tag{10}$$

In the asynchronous regime, contrarily, the instability condition turns out to be independent of this frequency: $H_c^2(\omega) = 2H_c^2$.

Actually, at the level of description we are proposing here, we can even address further questions concerning the eventual occurrence and dynamics of singularities, like orientation walls, defects, etc. in the pattern of reorientation of a sample subjected to experimental realizations of the Freedericksz transition under a rotating magnetic field. In particular, the possibility of detecting target-like or spiral patterns results specially exciting and is now under experimental consideration (Migler and Meyer, 1990; Kai *et al.*, 1992). Preliminary theoretical approaches have been also very recently ellaborated in terms of a perturbative ansatz based on two small parameters: the anisotropy of the elastic constants and the distance to the limiting conditions for synchronous rotations. Detailed calculations referring to the rotational dynamics of the preferred angle of orientation of a Brochard's wall show the coupling between the time scale of the synchronous rotation and that corresponding to the asynchronous one. These calculations will be published elsewhere (Sagues and Kai, 1992).

We would like to stress here that the present argument for pattern formations is still semi-microscopic one and cannot describe the macroscopic pattern dynamics consisting of groups of walls. In order to discuss its dynamics, one needs further reduction of variables on interface motions among domains, as done by Ohta *et al.* (1982) in general frame works.

## 3. Multiplicative Noise Effects

### *3.1 Magnetic Freedericksz instability*

The purpose is to elucidate the effect of fluctuations of the control parameter (external noise) on some statical and dynamical properties associated with the Freedericksz transition. The static behavior such as profiles of distribution functions in steady state under multiplicative noise has been discussed in Horsthemke *et al.* (1985). To render our analysis as simple as possible, we will refer to the case of the twist geometry and neglect any eventual inhomogeneities (bend modes) of the sort discussed previously. From the point of view of the external noise problem, the most significant feature is that the dynamical model is nonlinear (quadratic) in the magnetic field. This fact precludes the use of the simplest Gaussian white-noise assumption for the field fluctuations.

The nonlinear version of the dimensionless dynamical equation for the amplitude of the mode $m = 0$, $q_x = 0$ reads,

$$\partial_t \theta(t) = -U'(\theta) + \xi(t) = f(\theta) + h^2 g(\theta) + \xi(t), \tag{11a}$$

$$U(\theta) = \left(1 - h^2\right)\theta^2 / 2 + h^2\theta^4 / 8, \tag{11b}$$

$$f(\theta) = -\theta,$$

$$g(\theta) = \theta\left(1 - \theta^2 / 2\right), \tag{11c}$$

$$\tau_0 = \gamma_1 d^2 / K_{22}\pi^2 = \gamma_1 / \chi_a H_c^2, \tag{11d}$$

$$t = \tau_0^{-1} t'. \tag{11e}$$

We now consider the effect of external noise in the magnetic field. The experimental situation is similar to the one considered by Kai *et al.* (1979) and

Kawakubo *et al.* (1981). The theoretical analysis follow the line discussed by Sagues *et al.* (1985). We introduce an Ornstein-Uhlenbeck process, that is, a Gaussian process with zero mean and exponential correlation, to simulate the fluctuating part $w(t)$ of the magnetic field,

$$H \to H_0 + w(t) \tag{12a}$$

$$\langle w(t_1)w(t_2)\rangle = (D/\tau_N)\exp(-|t_1 - t_2|/\tau_N). \tag{12b}$$

In order to discuss the effect of the external fluctuations, the first problem is to find the equation for the probability distribution associated with the above Langevin equation. The difficulty comes from the nonlinearity of $w(t)$. An approximate Fokker-Planck representation which accounts for the essential effects of the external noise can be obtained in the limit $D \ll 1$, $\tau_N \ll 1$ with $D/\tau_N$ finite. Here $D$ and $\tau_N$ are the dimensionless noise intensity and correlation time respectively (Sagues and San Miguel, 1985). This approximation actually corresponds to the ordinary situation in which the noise evolves in a fast time scale, but has a finite strength measured by the integral of the spectral density $S(\omega)$: $D/\tau_N = \int d\omega S(\omega)$. This procedure is equivalent to a consistent Markovian limit which leads to a Fokker-Planck equation written as,

$$\partial_t P(\theta,t) = \partial_\theta\left[\overline{U}'(\theta) - \overline{D}g(\theta)g'(\theta)\right]P(\theta,t) + \partial_\theta^2\left[\overline{D}g^2(\theta) + \varepsilon\right]P(\theta,t), \tag{13a}$$

$$\overline{D} = D\left(4h^2 + D/\tau_N\right). \tag{13b}$$

Two different effects of the external noise may be readily noticed. The first and most important is a genuine consequence of the nonlinearity of the noise and amounts to a modification of the potential $U$ which is now replaced by $\overline{U}$, or equivalently by the substitution of $h^2$ by $\alpha^2 = \overline{h}^2 + D/\tau_N$. The second effect is the introduction of a state-dependent noise through the coupling function $g(\theta)$.

At this level two different consequences of the external noise can be analyzed: The shift of the instability point and the modification of the relaxation times. On what respects to the first question, and using the extrema of the stationary solution of the Fokker-Planck equation as an indicator, an straightforward calculation gives the threshold value for the Freedericksz transition as,

$$\overline{h}_F^2 = \left(1 - D/\tau_N + D^2/\tau_N\right)/(1 - 4D), \tag{14}$$

and also the assymptotically expanded form in powers of $\tau_N$

$$\overline{h}_F^2 = 1 - D / \tau_N + \left(4 - 3D / \tau_N\right) \approx 1 - D / \tau_N. \quad (15)$$

The dominant contribution, $-D/\tau_N$, implies that rapid external quadratic noise destabilizes the system lowering the threshold value of the Freedericksz instability. We estimate that typical realistic parameters of the noise may result in 5 percents variation in this threshold condition.

On what respects to the analysis of relaxation times, and taking into account the pure systematic effect manifested in the modification of the potential $U$, one immediately sees that the relaxation times towards the stable solution $\theta^2 = 0$ below the shifted instability decrease, whereas they increase when referring to the relaxation towards the stable distorted configuration $\theta^2 = 2(1-1/\alpha^2)$ above the modified Freedericksz instability.

*3.2 Dielectric instability (electrically induced Freedericksz instability)*

The pattern formation process due to the dielectric instability called the electrically induced Freedericksz instability is observed when an electric field is applied to the homeotropically oriented nematic sample with negative dielectric anisotropy $\varepsilon_a < 0$ (see Fig. 1). In this case, unlike in the electrohydrodynamic instability, an electrical conductivity is not necessary, i.e., no ionic current flows and only dielectrically induced deformation is taken into account. When the applied electric field E reaches to the threshold field $E_c^2 = \pi^2 K_{ii}/\varepsilon_0\varepsilon_a d^2$ ($K_{ii} = K_{33}$ in the case shown in Fig. 4) where $\varepsilon_0$ is the dielectric constant of vacuum, homeotropic orientation starts to deform in the bulk of a sample. The deviation from pure homeotropic orientation is represented by a tilt angle $\theta$. Energetically two angles $\pm\theta$ are identical and can be taken. Between them there is a boundary with singular orientation as shown in Fig. 4. If one applies a step field $E$ to the uniform homeotropic cell, due to initial fluctuation of the director, many domains with two different tilt angles in space transiently appear (Fig. 5). The similar transient hydrodynamic effect (back flow effect) with a magnetic field case has been already observed in the electrically induced Freedericksz instability (Buka *et al.*, 1989). This could be

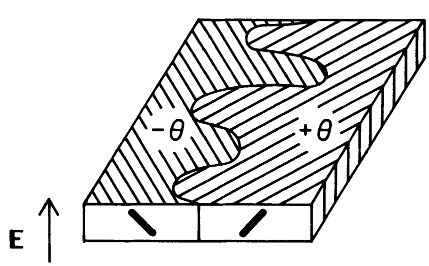


Fig. 4. Domain wall between different domains with two director orientations energetically identical againt an external field $E$.

(a)

(b)

(c)

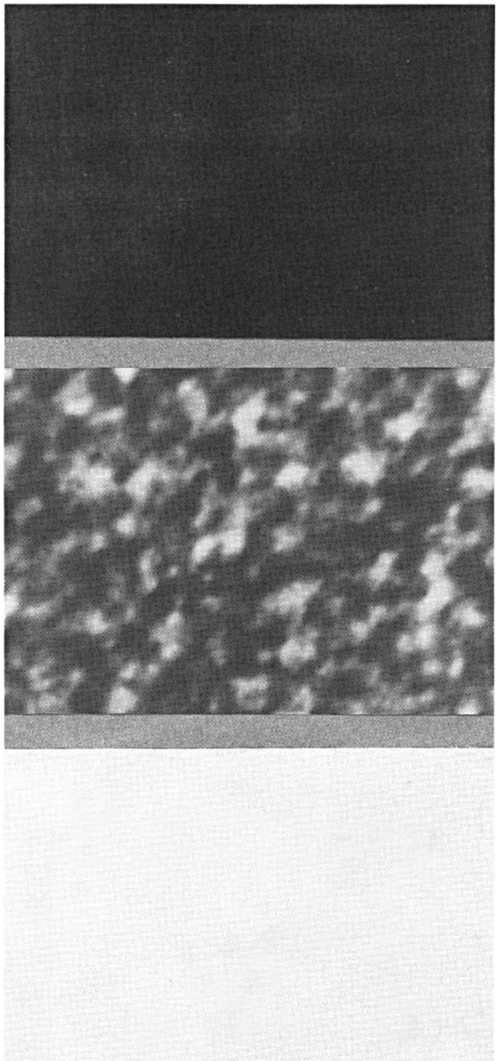

Fig. 5. Typical transient pattern in the electrically induced Freedericksz transition. (a) $t < 0$, (b) $0 < t$ (typically several seconds), (c) $t = \infty$.

explained by basically the same frame work described above replacing a magnetic field to an electric field. However, in actual experiments in the electric field ionic conduction has to be taken into account, i.e., additional sets of terms are needed (Winkler *et al.*, 1990). This again makes problems complicated. In this sense thus, the simple electrically-induced Freedericksz transition has been already well-understood (Blinov, 1979).

Here we will briefly discuss the non-trivial noise effect of the electrically induced Freedericksz transition in homeotropical oriented sample, the so-called multiplicative noise effect. A noise electric field is much more easily realized than a noise magnetic field. This is the reason why we study the electrically induced Freedericksz transition instead of the magnetically induced one. As the experimental set-up has been described elsewhere (Kai, 1989; Kai *et al.*, 1989), we will describe here only a preliminary result recently obtained. By replacing magnetic fields by electric fields in Eq. (15), one obtains equation for the electric field,

$$e_F^2 = 1 - D/\tau_N + D(4 - 3D/\tau_N) \sim 1 - D/\tau_N \tag{16a}$$

$$\langle \delta E(t)\delta E(0)\rangle / E_c^2 = D/\tau_N \times \exp(-t/\tau_N) = V_N^2 / V_c^2 \exp(-t/\tau_N). \tag{16b}$$

Here $\delta E(t)$ is a fluctuating (noise) electric field. Similarly the mean first passage time $T$ can be described by equation (Sagues and San Miguel, 1985),

$$T = \frac{1}{2}\ln\left(\frac{1}{2\varepsilon}\right) / \left(e_0^2 + D/\tau_N - 1\right), \tag{17}$$

$$e_0 = E/E_c = V/V_c.$$

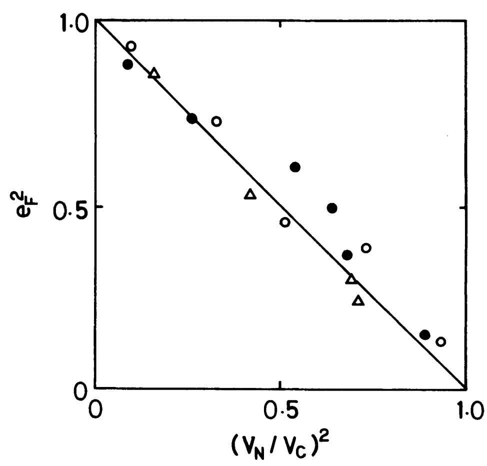


Fig. 6. Noise intensity $V_N$ dependence of threshold $e_F$ of the Freedericksz instability for various $\tau_N$ (Δ: 200 μs, o: 20 μs, o: 2 μs). The solid line is due to Eq. (16).

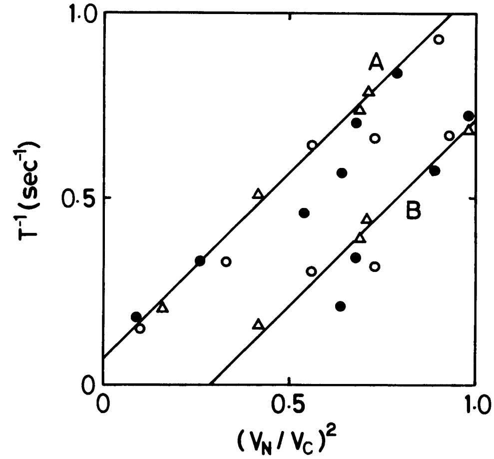


Fig. 7. Noise intensity dependence of the mean first passage time of the Freedericksz instability for various $\tau_N$ (Δ: 200 μs, o: 20 μs, o: 2 μs). The solid line indicates Eq. (17) with $V^2 = 18\ V^2$(A) and $12\ V^2$(B).

Here $e_0$ is the normalized field and the mean first passage time at an electric field $E$ in the absence of multiplicative noise respectively. $\varepsilon$ is the internal noise intensity and very large (~0.3) in this case. This may be due to the nonequilibrium noise enhancement (W. Schoepf, 1991). The experimental results are shown in Figs. 6 and 7 for shifts of the threshold and of the mean first passage time respectively. In the present study $T$ have been obtained by use of birefringence technique (Kai, 1989; Kai *et al.*, 1989). The solid lines are due to Eqs. (16) and (17) in Figs. 6 and 7 respectively. Both the shifts of a threshold and $T$ are on the same line for different $\tau_N$ respectively. Agreements between theoretical and experimental results are quite good. The investigation is still going on and its detail will be published elsewhere in near future.

### *3.3 Onset of electrohydrodynamics*

Multiplicative stochastic process in EHDI is much more complicated, because of spatial structures and hydrodynamic flows (Kai *et al.*, 1989). Detailed results experimentally obtained have been already described in Kai (1989) and Kai *et al.* (1989) and will be briefly summarized here. For the first bifurcation point in EHDI, noise influences as follows.

(1) When the correlation time $\tau_N$ of noise is much shorter than the characteristic time $\tau_c$ of a system, the threshold $V_c$ for the onset of Williams domain (roll convection) increases, i.e., $V_c$ is shifted up. However for $\tau_N \geq \tau_c$, $V_c$ decreases. Noise therefore can control both stabilization and destabilization.

(2) The structures at the first bifucation point ($V_c$) become more complicated

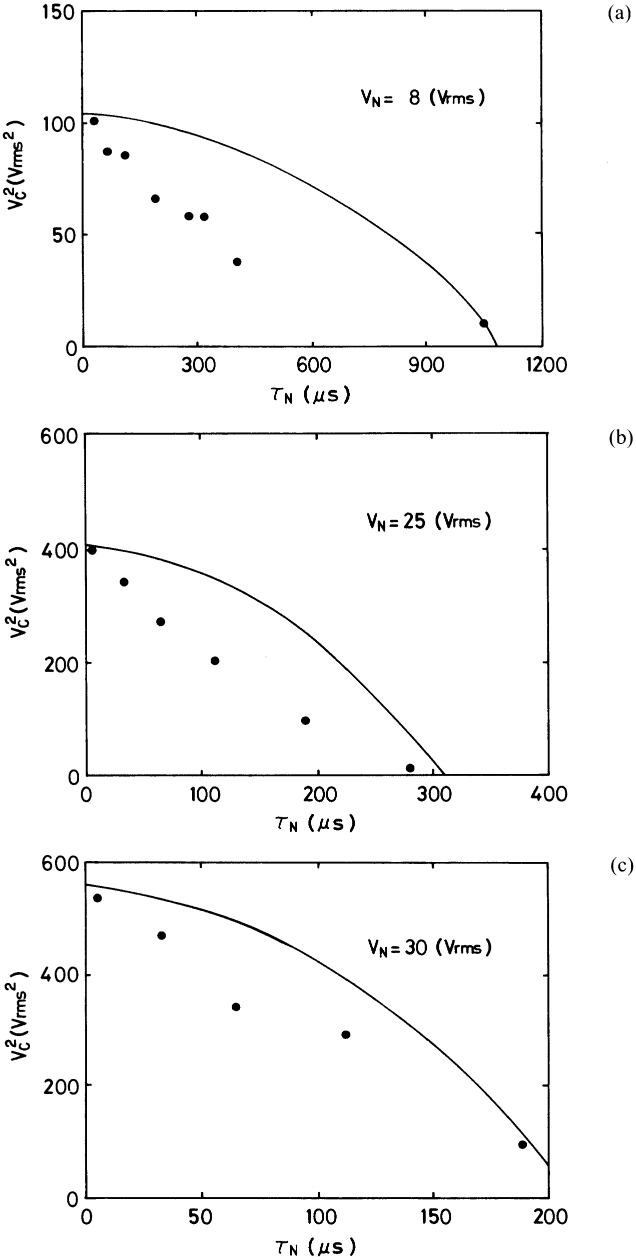


Fig. 8. Threshold shift for the onset of convection versus correlation time $\tau_N$ of noise in the electrohydrodynamic instability. The solid line is due to the Kuz and Wodkiewicz's analytical result. (a) $V_N = 8$ V, (b) 25 V, (c) 30 V.

when noise intensity increases at $\tau_N << \tau_c$. Finally, the direct transition to turbulence (DSM) is induced by noise (Brand *et al.*, 1985). This may be said that noise induces a multiple-dimension instability (Kai and Zimmermann, 1989).

(3) The noise intensity dependences of the threshold shifts are changed depending on the spatial structures which appear at $V_c$.

For these findings, theoretical works cannot explain all (Kai *et al.*, 1979; Kawakubo *et al.*, 1981; Kuz and Wodkiewicz, 1983; Müller and Behn, 1987, 1990). There are many contradictions from these theories in the experimental facts because most of them are based on linear theories, except one by Kai *et al.* (1979) where important flow effects however were neglected. They could only explain (1) the threshold shift to higher value for $\tau_N << \tau_c$ (Kawakubo *et al.*, 1981; Kuz and Wodkiewicz, 1983; Müller and Behn, 1987, 1990), (2) the threshold shift to lower value for $\tau_N \geq \tau_c$ (Müller and Behn, 1987, 1990). In order to explain the above experimental facts (2) and (3), we must rigorously take spatial degrees of freedom and nonlinear terms into account. This is usually difficult as well-known Swift-Hohenberg-type equations in Rayleigh-Bénard convection (Kramer *et al.*, 1989; Kai *et al.*, 1989).

The $\tau_N$-dependence of the threshold shift has been given analytically by theories (Kuz and Wodkiewicz, 1983; Müller and Behn, 1987). Some qualitative aspects obtained in experiments agreed with both theoretical results (Kai *et al.*, 1989). Noise used in our experiment is a Gaussian white noise, not a dichotomous noise (Müller and Behn, 1987) and $\tau_N$ can be changed by special electrical filters as already described elsewhere (Kai, 1989). Therefore, here we compare the experimental result of the $\tau_N$-dependence of the threshold shift with the theory obtained by Kuz and Wodkiewicz (KW) (1983). Figure 8 shows the threshold shift for various $\tau_N$. The solid line is due to KW's theory. The tendency decreasing the threshold shift with increase of $\tau_N$ well coincides with the experimental result but both slopes are quite different from each other. Namely experimental data shows rather concave while the theoretical curve shows convex shape. This indicates no applicability of the analytical result from a linear theory. More precise theory based on nonlinear equations will be required.

## 4. Conclusion

We have described our recent theoretical and experimental results of pattern formations in Freedericksz instability and its multiplicative stochastic processes as well as one in EHDI. In the Freedericksz instability the theoretical results well agree with experimental ones. Most of them could be described in the present theoretical frame works except an internal noise intensity. However, in a rotating magnetic field experimental results are not enough as well as theoretical consideration. Especially domain wall motions cannot explain yet and even dynamical equation for them are not obtained yet. We must think about further reduction of variables to look only macroscopic parts of the phenomenon. We are now achieving detailed experiments of soliton and spiral pattern formations under a rotating magnetic field most of which will be in future. In multiplicative noise effect in EHDI our experimental results show that as the correlation time $\tau_N$ increases, the threshold for the onset of convection decreases monotonically. The current theories based on a linear theory can predict such a tendency independent of either a Gaussian white or a dichotomous noise, but the dependence obtained experimentally is quite different from theoretical one. More complete theory would be required to explain the multiplicative noise effect in EHDI.

This work is in part supported by the Grand in Aid for Scientific Research from Ministry of Education, Science and Culture (No. 02452047). We would like to thank M. Zaharah for her assistance in the experimental measurements. One (F.S.) of the authors also thanks a research fellowship from the Kyushu Institute of Technology.

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