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Form Recognition Using Moment Invariants for Three Dimensional Perspective Transformations

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ABSTRACT

The invariant recognition of forms is important for many tasks. The purpose of this paper is to consider algebraic and moment invariants for perspective transformations. These are important because every lens system induces a perspective transformation. The approach consists of considering the non-linear perspective transformation in a higher dimensional, homogeneous space. In homogeneous space the perspective transformation is linear and algebraic invariant theory may be used to determine absolute algebraic and moment invariants. Examples are presented to demonstrate the theoretical approach. The significance of this work lies in the importance of invariant recognition for humans and machines.

1. INTRODUCTION

Recognition of the shape and form of objects in a scene is easily accomplished by human visual observations even if the object is translated, rotated, scaled, partially obscured, slightly distorted, or viewed in perspective. The invariant recognition of forms is important to humans for a variety of tasks. Even though variant recognition is also necessary for some tasks as illustrated by the differentiation of the characters b,d, and p, and problems such as dyslexia, invariant recognition is much more common. For machine vision invariant recognition of form is also important. For some problems, determining the position (translation transformation) or orientation (rotation transformation) of an object is of primary concern. For example, if a robot must pick up a known form object, the position and orientation of the object must be known in order to position and orient the robot hand. In other cases, the form of the object is of primary concern, not its position and orientation. Many examples of this situation occur in automatic inspection in which some property of the object must be measured and compared to a standard. The primary emphasis of this paper is on invariant measurements.

Several measurement exhibit invariant properties [1]. Of all the measurements which may be used for shape and form, moments have the most elegant mathematical theory of invariants. The use of two dimensional moment invariants was first proposed by Hu [2] in 1962 for character recognition. Two dimensional moment invariants of texture patterns in chest x-ray images were used as features for classification of coal workers pneumoconiosis by Hall, Crawford, and Roberts [3] in 1975. Two dimensional moment invariants were also used as features for aircraft identification by Dudani, Breeding, and McGee [4] in 1977. Several examples of numerical computations of two dimensional moment invariants are given in [5]. A theory of three dimensional moment invariants was developed by Sadjadi and Hall [6] in 1980. This theory will be extended in this paper.

The practical application of moments and invariants is illustrated by the use of moment computations in many available machine vision systems and of the recent introduction of an integrated circuit for computing moments.

The purpose of this paper is to present the extension of the moment invariant theory to include perspective transformations. The developement of perspective moment invariants is described and several examples are presented in Section 2. Finally, some conclusions and recommendations for further work are presented in Section 3.

2. PERSPECTIVE MOMENT INVARIANTS

The non-linear perspective transformation induced by a camera system can also be described by a linear transformation in homogeneous coordinates. The perspective transformation which corresponds to the pinhole camera model may be described by:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p & q & r & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \qquad (1)$$

where p,q,r are the reciprocals of the locations of the focal point (f_x,f_y,f_z) and w is an arbitrary constant.

The physical coordinates of a point are obtained from its homogeneous coordinates by dividing each of the first three components of the homogeneous coordinates by the fourth component.

The advantage of homogeneous coordinates is that a single transformation matrix can accomplish a full perspective transformation involving not only perspective but also rotation, translation, and scale.

The general perspective transformation is non-linear in non-homogeneous coordinates but in homogeneous coordinates the perspective transformation is linear and algebraic invariant theory can be used to determine absolute algebraic and moment invariants.

For a ternary quadratic in a general linear transformation the number of variables is three and the order is two, therefore, there are six independent relationships involving the six coefficients and parameters of the transformation. Since there are nine parameters of the transformation, there is no absolute invariant except the discriminent.[6] For a quaternary quadratic in a general linear transformation the number of variables is four and order is two, there are ten independent relationships involving the ten coefficients and parameters of the transformation. Since there are 16 parameters of the transformation, again there is no absolute invariant except the discriminant.

Let us first consider some two dimensional examples. In these cases the perspective transformation is described in three dimensional homogeneous coordinates.

Example 1
Consider the following example in two dimensions given the ternary quantic of order two;

$$f = a_1x^2 + a_2y^2 + a_3w^2 \tag{2}$$

The perspective transformation is described by

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} \tag{3}$$

The transformed quantic is given by;

$$f(x',y',w') = (a_1 + a_3)\, x'^2 + a_2y'^2 + a_3w'^2 - 2a_3x'w' \tag{5}$$

Since the Hessian is an invariant, the Hessian determinant is computed before the transformation as

$$H = \begin{vmatrix} 2a_1 & 0 & 0 \\ 0 & 2a_2 & 0 \\ 0 & 0 & 2a_3 \end{vmatrix}$$

$$= 8\, a_1a_2a_3 \tag{6}$$

After the transformation, the Hessian determinant is

$$H' = \begin{vmatrix} 2(a_1 + a_3) & 0 & -2a_3 \\ 0 & 2a_2 & 0 \\ -2a_3 & 0 & 2a_3 \end{vmatrix}$$

$$= 8\, a_1a_2a_3 \tag{7}$$

Consider the same ternary quadratic f with a change of notation for the coefficients, i.e.

$$f = a_{200}x^2 + a_{020}y^2 + a_{002}w^2 \tag{8}$$

The discriminant, Δ, is also an invariant,

$$\frac{\partial f}{\partial x} = 2a_{200}\, x$$

$$\frac{\partial f}{\partial y} = 2a_{020}\, y$$

$$\frac{\partial f}{\partial w} = 2a_{002}\, w$$

$$\Delta = 8 \quad a_{200}a_{020}a_{002} \tag{9}$$

Another invariant can be formed by using the Hessian

$$H = 8\begin{vmatrix} a_{200} & 0 & 0 \\ 0 & a_{020} & 0 \\ 0 & 0 & a_{002} \end{vmatrix} \quad (10)$$

Hence the Hessian is the same as the discriminant in this case. According to the fundamental theorem of moment invariants, there exists the following relation for a perspective transformation;

$$\mu_{200}\mu_{020}\mu_{002} = \mu'_{200}\mu'_{020}\mu'_{002} \quad (11)$$

Hence,

$$I_1 = \mu_{200}\mu_{020}\mu_{002} \quad (12)$$

is a moment invariant form.

Example 2.
Consider the quadratic form

$$f = a_1x^2 + a_2y^2 + a_3w^2 + 2b_1xy + 2b_2yw + 2b_3xw \quad (13)$$

$$f(x',y',w') = (a_1 + a_3 - 2b_3)\ x'^2 + a_2y'^2 + a_3w'^2 + 2(b_1-b_2)x'y'$$
$$+2b_2y'w' + 2(b_3 - a_3)x'w' \quad (14)$$

The discriminent Δ is also the same as the Hessian.
Again, according to the fundamental theorem of moment invariants, there exists the following moment invariant relation for a perspective transformation;

$$\mu_{200}\mu_{020}\mu_{002} + 2\mu_{110}\mu_{011}\mu_{101} - \mu_{200}\mu^2_{011} - \mu_{020}\mu^2_{101} - \mu_{002}\mu^2_{110}$$
$$= \mu'_{200}\mu'_{020}\mu'_{002} + 2\mu'_{110}\mu'_{011}\mu'_{101} - \mu'_{200}\mu'^2_{011}$$
$$- \mu'_{020}\mu'^2_{101} - \mu'_{002}\mu'^2_{110} \quad (15)$$

Hence,

$$I_1 = \mu_{200}\mu_{020}\mu_{002} + 2\mu_{110}\mu_{011}\mu_{101} - \mu_{200}\mu^2_{011} - \mu_{020}\mu^2_{101}$$
$$-\mu_{002}\mu^2_{110} \quad (16)$$

Example 3.
Consider the following quaternary quantic of order two;

$$f = a_1x^2 + a_2y^2 + a_3z^2 + a_4w^2 \quad (17)$$

and the perspective transformation described by

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \quad (18)$$

The transformed quantic is given by;

$$f(x',y',f',w') = (a_1 + a_4)\, x'^2 + a_2y'^2 + a_3z'^2 + a_4w'^2 - 2a_4x'w' \quad (19)$$

Since the Hessian is an invariant, the Hessian determinant is computed before the transformation.
Also, the Hessian is the same as the discriminant.
According to the fundamental theorem of moment invariants, there exists the following moment invariant relation for a perspective transformation;

$$\mu_{2000}\mu_{0200}\mu_{0020}\mu_{0002} = (\mu'_{2000} + \mu'_{0002})\mu'_{0200}\mu'_{0020}\mu'_{0002} \quad (20)$$

Hence,

$$I_1 = \frac{(\mu_{2000} + \mu_{0002})\mu_{0200}\mu_{0020}\mu_{0002}}{\mu_{2000}\mu_{0200}\mu_{0020}\mu_{0002}} \quad (21)$$

is a moment invariant form.
Example 4.
Consider the quadratic form

$$f = a_1x^2 + a_2y^2 + a_3z^2 + a_4w^2 + 2b_1xy + 2b_2xz + 2b_3xw + 2b_4yz + 2b_5yw + 2b_6zw \quad (22)$$

From (18), the transformed quantic is

$$f(x',y',z',w') = (a_1 - 2b_3)\, x'^2 + a_2y'^2 + a_3z'^2 + a_4w'^2 + (2b_1 - 2b_5)x'y' + (2b_2 - 2b_6)x'z' + 2b_3x'w' + 2b_4y'z' + 2b_5y'w' + 6b_6z'w' \quad (23)$$

The Hessian is the same as the discriminant. According to the fundamental theorem of moment invariants, there exists the following moment invariant relation for a perspective transformation;

$$I_1 = \frac{\Delta\mu'}{\Delta\mu} \quad (24)$$

where

$$\Delta\mu = \begin{vmatrix} \mu_{2000} & \mu_{1100} & \mu_{1010} & \mu_{1001} \\ \mu_{1100} & \mu_{0200} & \mu_{0110} & \mu_{0101} \\ \mu_{1010} & \mu_{0110} & \mu_{0020} & \mu_{0011} \\ \mu_{1001} & \mu_{0101} & \mu_{0011} & \mu_{0002} \end{vmatrix}$$

$$\Delta\mu' = \begin{vmatrix} \mu_{2000} - 2\mu_{1001} & \mu_{1100} - \mu_{0101} & \mu_{1010} - \mu_{0011} & \mu_{1001} \\ \mu_{1100} - \mu_{0101} & \mu_{0200} & \mu_{0110} & \mu_{0101} \\ \mu_{1010} - \mu_{0011} & \mu_{0110} & \mu_{0020} & \mu_{0011} \\ \mu_{1001} & \mu_{0101} & \mu_{0011} & \mu_{0002} \end{vmatrix}$$

is a moment invariant form.

Example 5.

In the general case, the perspective transformation is described by four dimensional homogeneous coordinates.

We will now develop the invariant moments for perspective transformations.

Consider the quadratic form

$$f = a_1x^2 + a_2y^2 + a_3z^2 + a_4w^2 + 2b_1xy + 2b_2xz + 2b_3xw$$
$$+ 2b_4yz + 2b_5yw + 2b_6zw \qquad (25)$$

and the perspective transformation described by (1)

The transformed quantic is

$$\begin{aligned} f(x',y',z',w') = &(a_1 - 2b_3p + a_4p^2)x'^2 + (a_2 - 2b_5q + a_4q^2)y'^2 \\ &+ (a_3 - 2b_6r + a_4r^2)\, z'^2 + a_4w'^2 \\ &+ (2b_1 - 2b_5p + 2a_4pq - 2b_3q)x'y' \\ &+ (2b_2 - 2b_3r - 2b_6p + 2a_4pr)x'z' \\ &+ (2b_3 - 2a_4p)x'w' \\ &+ (2b_4 - 2b_5r - 2b_6q + 2a_4qr)y'z' \\ &+ (2b_5 - 2a_4q)y'w' + (2b_6 - 2a_4r)z'w' \end{aligned} \qquad (26)$$

The Hessian is the same as the discriminant. According to the fundamental theorem of moment invariants, there exists the following moment invariant relation for a perspective transformation;

$$I_1 = \frac{\Delta\mu'}{\Delta\mu} \qquad (27)$$

where

$$\Delta\mu = \begin{vmatrix} \mu_{2000} & \mu_{1100} & \mu_{1010} & \mu_{1001} \\ \mu_{1100} & \mu_{0200} & \mu_{0110} & \mu_{0101} \\ \mu_{1010} & \mu_{0110} & \mu_{0020} & \mu_{0011} \\ \mu_{1001} & \mu_{0101} & \mu_{0011} & \mu_{0002} \end{vmatrix}$$

$$\Delta\mu' = \begin{vmatrix} (\mu_{2000} - 2p\mu_{1001} + p^2\mu_{0002}) & (\mu_{1100} - p\mu_{0101} + pq\mu_{0002} - q\mu_{1001}) & (\mu_{1010} - r\mu_{1010} - p\mu_{0011} + pr\mu_{0002}) & (\mu_{1001} - p\mu_{0002}) \\ (\mu_{1100} - p\mu_{0101} + pq\mu_{0002} - q\mu_{1001}) & (\mu_{0200} - 2q\mu_{0101} + q^2\mu_{0002}) & (\mu_{0110} - r\mu_{0101} - q\mu_{0011} + r^2\mu_{0002}) & (\mu_{0101} - q\mu_{0002}) \\ (\mu_{1010} - r\mu_{1010} + p\mu_{0011} + pr\mu_{0002}) & (\mu_{0110} - r\mu_{0101} - q\mu_{0011} + qr\mu_{0002}) & (\mu_{0020} - 2r\mu_{0011} + r^2\mu_{0002}) & (\mu_{0011} - r\mu_{0200}) \\ (\mu_{1001} - p\mu_{0002}) & (\mu_{0101} - q\mu_{0002}) & (\mu_{0011} - r\mu_{0200}) & (\mu_{0002}) \end{vmatrix}$$

is a moment invariant form.

A line in original coordinates is shown in Fig. 1. This line before transformation is mapped to a line parallel to the original line as shown in the homogeneous coordinates of Fig. 2. After perspective transformation, the line shown in Fig. 3 is not parallel to the original line. The image of a line is uniquely determined by the images of its end points since a perspective transformation maps a line into a line.

4. CONCLUSIONS

In this paper we have considered algebraic and moment invariants for perspective transformations. The procedure involved converting the non-linear perspective transformation in an original space to a linear transformation in homogeneous space. Examples were also presented which demonstrate the technique.

Futher research is needed to: expand the theoretical basis of moment invariants; relate the invariants to known geometrical properties; relate the invariants to human recognition; use the invariants for recognition in a variety of applications.

REFERENCES

[1] E.L. Hall, Computer Image Processing and Recognition, Academic Press, 1979.

[2] M.K. Hu, "Visual pattern recognition by moment invariants," IRE Trans. Inform. Theory, vol. IT-8, pp. 179-187, Feb. 1962.

[3] E.L. Hall, W.O. Crawford, Jr., and F.E. Roberts, "Computer classification of pneumocpmopsis from radiographs of coal worker, "IEEE Trans. Biomed. Eng., vol. BME-22, pp. 518-527, Nov. 1975.

[4] S.A. Dudani, K.F. Breeding, and R.B. McGee, "Aircraft identification by moment invariants," IEEE Trans. on Computers, vol. C-26, no, 2. pp. 39-45, October, 1977.

[5] F.A. Sadjadi and E.L. Hall, "Numerical computation of moment invariants for scene analysis," in Proc. IEEE Conf. on Pattern Recognition and Image Processing, Chicago, IL, 1978.

[6] F.A. Sadjadi and E.L. Hall, "Three-dimensional moment invariant," IEEE Trans. Pattern Anal. Machine Intell., vol. PAMI-2., pp. 127-136, Mar. 1980.

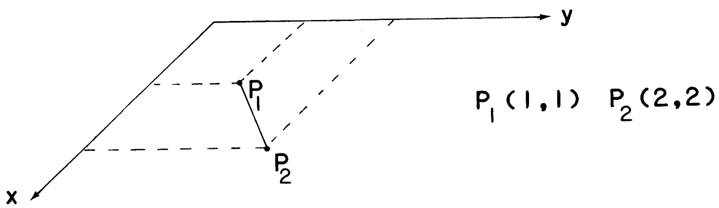

Figure 1. Original coordinate.

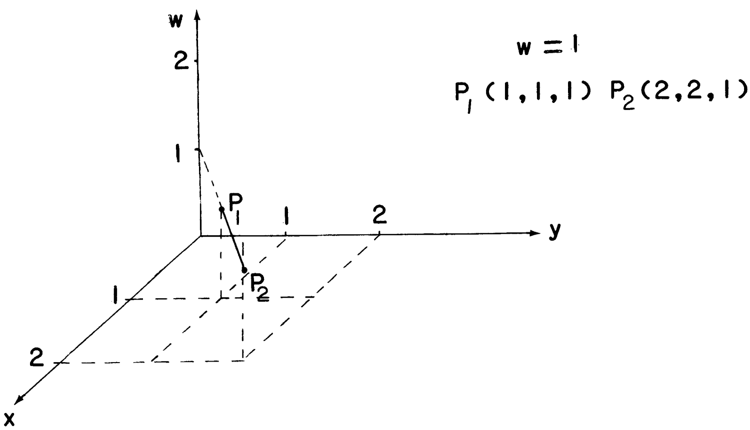

Figure 2. Homogeneous coordinate (before transformation).

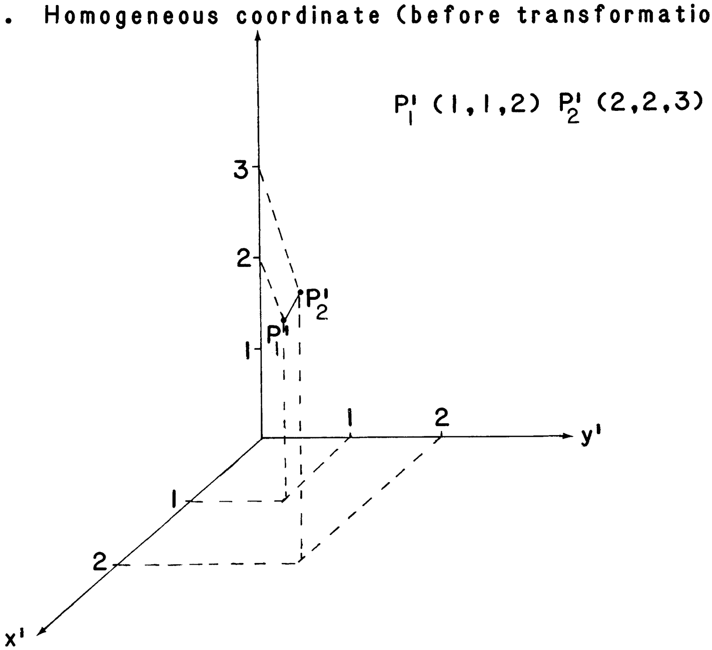

Figure 3. Homogeneous coordinate (after transformation).

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Q: In 2 dimensional cases, we can observe all boundaries of an object; the moments of an object can be obtained easily using the boundaries. Thus, the moments are important features for the recognition tasks.

On the other hand, in 3 dimensional cases, we can observe only one part of the boundaries of an object and the observable shapes are dependent on the viewers directions. It is very difficult to determine moments of an object from the partial boundaries. Thus, it is essentially impossible to use moments for 3-D object recognition.

How do you deal with this problem in your case? (K. Ikeuchi)

A: I believe we can deal with this situation much like humans do. I observe the front of the head but infer the existence of the back of the head. Three dimensional measurments from a portion of an object may be used to match a model of the entire object. The computed model match would then provide a "good" assumption about the entire object.

Q: Dr. Hall, are you aware of the work related to the influence of the "sectioning transformation" on invariant moments of 3-D objects? (H. J. Gundersen)

A: No. I am not sure what you mean by "sectioning transformation". If you mean cutting a section through a 3D object to obtain a 2D planar section, then I would have to do some work to relate this to moment invariants. If the sectioning involves a continuous integral, I suspect there is a relation much like the one we use in computed tomography.

Q: I enjoyed the paper because of the mathematics, but that's because I'm by myself a mathmatician. Would you please be sokind and give me an example for the use of the moment invariance for an 3-D object recognition for better understanding how this method works in various applications. (D. Koenig)

A: Moments are now commonly used as recognition features. For example many machine vision systems in the U.S. use moments of "blobs" for inspection parameters. Moment invariants have been used for character recognition, chest x-ray analysis, aircraft recognition, and for many matching. We have used 3-D moment invariants for recognizing objects such as cups and footballs. I believe a group at General Motors are also using these for automobile inspection.

C: Maxwell, I think, showed that the decomposition of an object as a series of moments is formally equivalent to decomposition as spherical harmonics, so that invariants of moments are equivalent to concepts such as sphere size ellipticity. The question of object recognition is equivalent to that of characterizing molecular shape so that progress in these two areas will be of mutual importance. (A. Mackay)

C: In using moments in Pattern Recognition, each moment itself may be used instead of invariants, if we can normalize moment values concerning the related distortion. I would like to expect your comment on comparison of two approaches A and B.

(J. Toriwaki)