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Self-Induced Vibration of an Evaporating Drop

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Abstract. A review is given on the experiment and the theory about a self-excited vibration of a rapidly evaporating drop of liquefied-gas, which is placed on a horizontal floor with room temperature. The drop showed modes with polygonal shapes in the plane view and made transitions to other modes as it reduced size through evaporation. The process of mode transition did not depend on the temperature of the surrounding air. Some theoretical investigations of this phenomenon are explained. These researches provide a new aspect of pattern dynamics of a highly inequilibrium system.

1. Introduction

Let a drop of liquid nitrogen or oxigen (whose boiling temperatures are –195 and –173°C, respectively) on a horizontal plate with room temperature, then it deforms to a thin circular disk owing to the gravity. The drop is in an overheated state, and the vapor coming from the lower surface of the drop has an enough pressure to levitate the drop from the plate. According to our observation (Adachi and Takaki, 1984) such a drop showed a characteristic vibration, where the thickness of the drop remained constant and its plane contour oscillated around a circular shape with relatively large amplitude. It could move without friction from the plate by the presence of a vapor sheet under it. Number n of waves (or vertices) along the contour changed according to the drop size. This vibration continued until it disappeared through evaporation.

In spite of the fact that it is commonly observed in laboratories, mechanism of

vibration is not understood yet. A similar experiment was made before (Holter *et al.*, 1952), but there seems to be no other reports on research of this phenomenon. A general review of this phenomenon is given by one of the present authors (Takaki, 1990).

Analyses of small amplitude vibrations of a spherical drop and a circular column are made by Rayleigh (1879, 1902) and the following formula for frequencies ω_d and ω_c are obtained:

$$\omega_d^2 = \frac{(n-1)n(n+2)\sigma}{r_0^3\rho}, \quad \text{for a spherical drop,} \tag{1}$$

$$\omega_c^2 = \frac{(n-1)n(n+1)\sigma}{r_0^3\rho}, \quad \text{for a circular column,} \tag{2}$$

where n is the degree of spherical harmonic function for the sphere case and the number of waves for the column case, σ is the surface tension coefficient, r_0 is the mean radius, and ρ is the liquid density.

For the case of a flattened drop as in our experiment, the present authors (Takaki and Adachi, 1985) developed a linear theory to predict its frequency based on the invicsid hydrodynamics and the shallow water approximation. Result of the theory agrees well with our experiment (Adachi and Takaki, 1984). However, the linear analysis is not concerned with such problems as determination of amplitude, mechanism of excitation of vibration or that of mode transition.

With a purpose to investigate mechanism of vibration, the present authors recently made an experiment to change the surrounding temperature within –90°C and 200°C and to observe vibrational modes.

In this paper, a brief summary of our past observations and analyses is given, and based on these researches, an aspect of pattern formation phenomena in highly inequilibrium state is suggested.

2. Experiment

2.1 Mode selection and vibrational frequency

A drop of liquid-nitrogen, liquid-oxygen or liquid-argon with volume 0.1–0.2 cm^3 was put carefully at the center of a slightly concave lens with radius of curvature 10 cm, which was placed horizontally and kept at room temperature. Motion of the drop was recorded by the use of 16 mm cine camera or video camera from above. The lens was used in order to keep the drop within the frame of the camera. Immediately after it was put on the lens, the drop began vibrating, where the thickness of the drop was nearly constant (1.5 mm) but the contour in the plane view showed a standing wave, as is shown in Fig. 1. Period of vibration was O(30 ms)

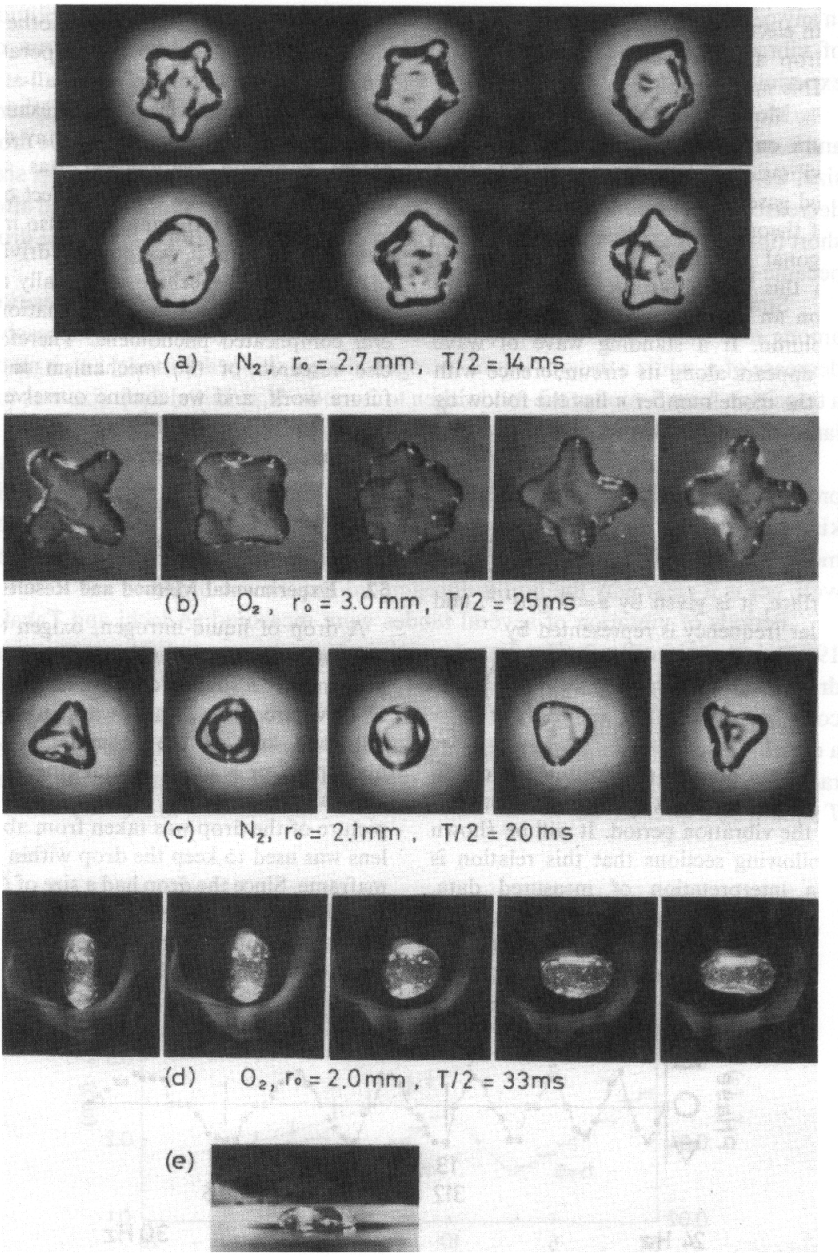

Fig. 1. Plane views of drops in polygonal vibration, taken by a high-speed camera. (a), (c), (d) with 400 frames/s, (b) with 4000 frames/s, (e) a side view of 4-mode. (from Adachi and Takaki (1984)).

in any case. This type of motion is called here a polygonal vibration. The amplitude of vibration remained constant and had nearly the same value at any run of experiment.

Mode of vibration is indicated by the number n of waves along the periphery and is called n-mode. A mode was selected almost uniquely depending on the drop size, i.e., for a smaller drop the mode number n was smaller. As the drop size decreased through evaporation, the drop motion changed to a random one and after short time recovered another regular mode with smaller n. After 2-mode the drop became nearly spherical and disappeared.

Strange to say, there was a clear distinction in the procedure of mode transition among materials. Drops of liquid-oxygen and liquid-argon showed a successive decrease of n, while a drop of liquid-nitrogen always skipped 4-mode, i.e., it made a transition from 5-mode to 3-mode, then to 2-mode. If an initial drop size was too large, the drop showed either random motion or splitting to small drops.

Quantitative measurement of mode transition was made by the use of a image processing technique (Takaki *et al.*, 1986b, 1989), and it was found that a certain kind of nonlinear interaction among several modes was taking place during the mode transition. However, the mechanism of mode transition is not understood well, nor is it clear why the liquid-nitrogen did not show 4-mode.

Periods of vibration of several modes were measured (Adachi and Takaki, 1984) as shown in Fig. 2. The data show clear dependence on $r_0^{3/2}$ (r_0 is the mean drop radius in the plane view). Although the mechanism of fluid motion is complicated, the dependence of the period on the drop radius is obtained easily by a dimensional analysis. Suppose the dynamics is determined by the density ρ, the radius r_0, the surface tension coefficient σ and the liquid density ρ, then the period T should be expressed as

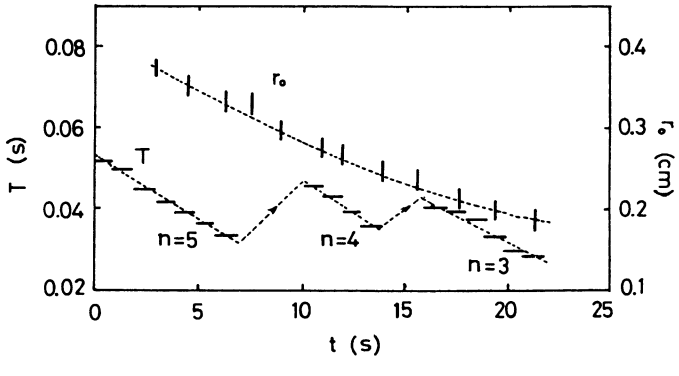

Fig. 2. Variations of mean radius r_0 and period of vibration T during one run of experiment for liquid-oxygen. (from Adachi and Takaki (1984)).

$$T = \text{const.}\left(r_0^3 \rho / \sigma\right)^{1/2}, \tag{3}$$

where the constant may depend on the mode number n.

3. Theoretical Approach

Theoretical study is immediately faced with difficulty coming from the complicated dynamics. However, certain simplified analysis may be possible as explained below.

3.1 Static state

First, a static state is treated and the vibration is looked upon as a perturbation on this static state. In the static state the drop is a circular disk, whose thickness is estimated from a balance of the gravity and the surface tension at the peripheral region.

Let us denote the thickness by h_0, and assume that the radius of curvature at the periphery in the vertical cross section is $h_0/2$. Then the balance of the gravity and the surface tension leads to the following relation:

$$\frac{1}{2}\rho g h_0 \left(\text{averaged static pressure}\right) = \frac{\sigma}{h_0 / 2}. \tag{4}$$

By substituting material values of liquefied gases, we have h_0 = 1.8 mm, which agrees with measurement (1.5 mm).

A more precise argument can be made by considering also the radius of curvature R_0 in the plane view (Takaki and Adachi, 1985), and the result is

$$\frac{1}{2}GH^2 = 2 + \frac{1}{H^{-1} + \beta}, \quad \text{where } G = \frac{\rho g R_0^2}{\sigma}, \; H = \frac{h_0}{R_0}, \tag{5}$$

and β is given in Eq. (7). The parameter G indicates the importance of the gravity relative to the surface tension, and H is the aspect ratio. If the drop is thin, i.e., $H << 1$, then we have

$$G >> 1, \; GH^2 = 4, \; GH = O\left(H^{-1}\right) >> 1. \tag{6}$$

3.2 Linear theory of vibration

Normal mode of small amplitude vibration is treated to obtain vibration frequency. Motion of liquid is governed by the Navier-Stokes equation, the

continuity equation and the boundary conditions at the liquid surface. Assumptions made here are as follows:

(1) The aspect ratio h_0/R_0, where R_0 is the mean radius in the plane view, is sufficiently small. The fluid motion is looked upon as a surface wave on the liquid layer and the shallow water theory is applied. Depth of the liquid is denoted by h(r, θ, t).

(2) The contour of drop shape in the plane view is denoted by $R^*(\theta, t)$. Boundary conditions for shallow water motion are applied at a modified boundary $r = R(\theta, t)$, which is defined so that the semi-circle in the vertical cross section has the same area as the rectangular region ending at the modified boundary (see Fig. 3), i.e.,

$$R^*(\theta,t) = R(\theta,t) + \beta h(R,\theta,t), \quad \text{where } \beta = \frac{1}{2} = \frac{\pi}{8} = 0.107. \tag{7}$$

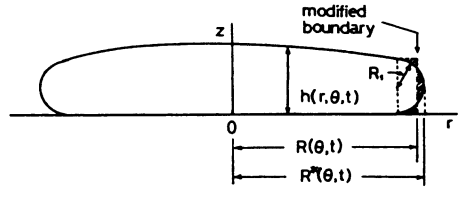

Fig. 3. Assumed geometry and definitions of variables. (from Takaki and Adachi (1985)).

(3) The lower boundary of the liquid layer remains horizontal and suffers no frictional force from the floor.

(4) The liquid flow is incompressible and inviscid.

(5) Effect of evaporation is neglected, hence all material parameters and the drop volume is constant during vibration.

Assumptions (1), (2) and (3) do not contradict our observations, while (4) and (5) need some examination. Typical Reynolds number for vibrating drop is $O(100)$, hence the dynamics can be analysed in terms of inviscid flow. Both effects of damping due to viscosity and driving force due to evaporation belong to long time-scale phenomena, and can be neglected in the analysis of vibration. The volume change during one period of vibration is also negligible.

Basic equations in the shallow water theory are the continuity equation and the momentum equation, which are written in the cylindrical coordinates as follows:

$$\frac{\partial h}{\partial t} + \frac{1}{r}\frac{\partial (rhu_r)}{\partial r} + \frac{1}{r}\frac{\partial (hu_\theta)}{\partial \theta} = 0, \tag{8}$$

$$\frac{\partial}{\partial t}(\rho h\boldsymbol{u}) + \frac{\partial}{\partial r}(u_r \rho h\boldsymbol{u}) + \frac{1}{r}\frac{\partial}{\partial \theta}(u_\theta \rho h\boldsymbol{u}) = -\mathrm{grad}P, \tag{9}$$

where $\boldsymbol{u} = (u_r, u_\theta)$ is the velocity vector in the horizontal direction, $P = \rho g h^2/2 - \sigma\Delta h$ is the pressure integrated along the vertical direction and ρ is the liquid density.

The pressure condition at the modified boundary is

$$P = h\sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right), \quad \text{at } r = R(\theta, t), \tag{10}$$

where $R_1 = h(R(\theta, t), \theta, t)/2$ is the radius of semi-circle at the periphery in the vertical cross section, and R_2 is the radius of curvature of the plane contour expressed by $r = R(\theta, t)$. The kinematical condition is written as

$$u_r = u_\theta \frac{1}{r}\frac{\partial R}{\partial \theta} + \frac{\partial R}{\partial t}, \quad \text{at } r = R(\theta, t). \tag{11}$$

Procedure of normal mode analysis of vibration is given briefly below. Let r and t be normalized by R_0 and $R_0 = \sqrt{\rho R_0^3 / \sigma}$, respectively, and the same notations are used for normalized quantities. Perturbations of variables (denoted with tilde) are introduced as

$$h = R_0 H\left\{1 + \tilde{h}(r, \theta, t)\right\}, \quad R = R_0\left\{1 + \tilde{R}(\theta, t)\right\}$$
$$u_r = \frac{R_0}{T_0}\frac{\partial \tilde{\phi}}{\partial r}, \quad u_\theta = \frac{R_0}{T_0}\frac{\partial \tilde{\phi}}{\partial \theta}, \tag{12}$$

where $\tilde{\phi}$ is the velocity potential. For vibration of n-mode, the variables are written as

$$\left(\tilde{\phi}, \tilde{h}, \tilde{R}\right) + \left(\hat{\phi}(r), i\hat{h}(r), i\hat{R}\right)\exp(in\theta + i\Omega t), \tag{13}$$

where quantities with hat are real.

Then, from the governing equations (8)–(11) we obtain, after some manipulations,

$$\left(\Delta_n + w^2\right)\hat{\phi} = 0, \quad \text{where } \Delta_n = \frac{1}{r}\frac{d}{dr}r\frac{d}{dr} - \frac{n^2}{r^2}, \quad w^2 = \Omega^2 / GH, \tag{14}$$

$$\left(n^2 - 1\right)\hat{\phi}' = w^2\left\{\left(1+\beta H\right)^2 GH - \left(1+\beta H\right) - \left(n^2 - 1\right)\beta H\right\}\hat{\phi}, \quad \text{at } r = 1, \tag{15}$$

where ϕ' is the derivative of ϕ. In deriving these results we have neglected terms of $O(H^2)$.

Solution of Eq. (14) free from singularity at the origin is the Bessel function of the first kind, i.e.,

$$\hat{\phi} = AJ_n(wr), \tag{16}$$

where A is an arbitrary constant. Then, Eq. (15) leads to an eigenvalue equation for Ω, as follows:

$$\frac{J_n'(w)}{J_n(w)} = \frac{GHw\left\{\left(1+2\beta H\right) - (GH)^{-1} + O\left(H^2\right)\right\}}{N^2 - 1}$$

where the prime denotes derivative. After neglecting terms of $O(H^2)$ again, we have

$$\Omega^2 = n\left(n^2 - 1\right) / \left\{\left(1+2\beta H\right) + \frac{n-3}{2GH}\right\}. \tag{17}$$

Let us compare this result with experimental data (Adachi and Takaki, 1984). Experimental values of H were within $0.38 < H < 0.52$ for liquid oxigen in 6-mode vibration. Values of $\sqrt{\sigma / \rho}$ were chosen as 2.9, 3.2, 2.9 ($cm^{3/2}/s$) for liquid-nitrogen, -oxigen and -argon, respectively, in order to get best fit to experimental data. These values are smaller than those at the boiling points, 3.4, 3.5, 3.1, respectively, by several percent.

Figure 4 shows comparison of theory with experiment along with Rayleigh's formulae (1) and (2). The narrow width in theoretical values comes from those of H used for their evaluations. The agreement of the present theory with experiment is better than Rayleigh's results, especially for larger n. On the other hand, the formula (1) agrees best to data of 2-mode. This result is convincing, because the drop in 2-mode was small enough to have a spherical form rather than a thin liquid layer assumed in the theory.

The above analysis gives a good prediction of the vibrational frequency for relatively higher mode number. However, it is still not clear how the drop can

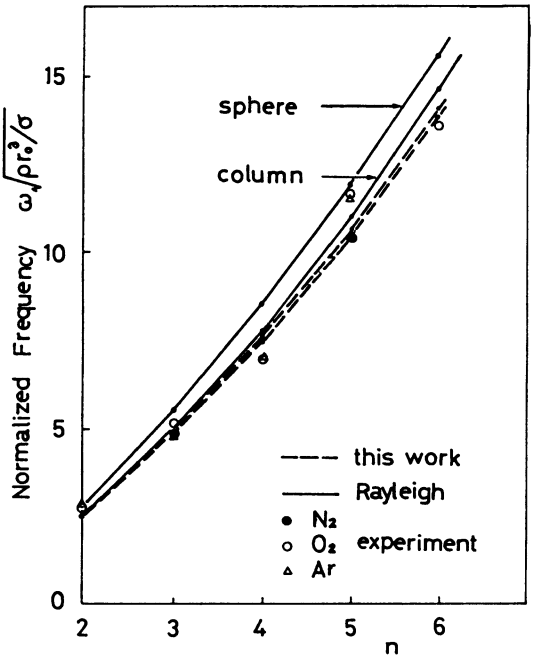

Fig. 4. Comparison of theoretical values of frequency with experiment. Rayleigh's formulae are also shown. Width in the theoretical result comes from that in experimental values of H. (from Takaki and Adachi (1985)).

maintain vibration, how a particular mode is selected or why the mode selection depends on material.

3.3 Model simulation of mode transition

In order to get a hint on these questions, a computer simulation is made by the present authors based on a simple model showing self-excited oscillation (1986a, 1989). The model includes a term with parametric modulation of the frequency, the decrease of drop size, a nonlinear interaction of modes and a dissipative effect. Let $u(\theta, t)$ denote the displacement of drop contour and be expressed by a Fourier series,

$$u = \Sigma u_n(t)\exp(in\theta). \tag{18}$$

Let us normalize length and time by the initial drop radius r_0 and the time scale T_0 (see Subsection 3.2). The assumed equation for model simulation is

$$\ddot{u}_n + \gamma\dot{u}_n + NL_n(u) + \omega_0^2\{1 + \alpha\cos(2\omega t)\}u_n = 0, \quad (n = 2, \ldots, 6) \tag{19}$$

where ω_0 and $NL_n(u)$ are given as follows:

$$\omega_0 = 2.5(n-1)(1+\beta t)^{3/2}, \quad \beta = 0.05,$$

$$NL_n(u) = n\text{– th Fourier component of } (\varepsilon u\dot{u}).$$

The parameters ε and α are constants, the dot over u denotes the time derivative and ω is fixed at 10. Interpretation of each term in Eq. (27) will be found in the paper by Takaki *et al.* (1989). For a given initial condition variations of several modes are calculated by solving Eq. (19), and the summary of results is given here.

In relatively narrow region of (α, ε) successive appearances of several modes are observed, where the mode number decreases. For α much larger than about 0.3 or ε > 17, the drop always shows burst. For opposite conditions, it tends to stop vibrating. These results agree with observation at least qualitatively and several suggestions are obtained. However, true understanding would not be possible, until hydrodynamical analysis considering effect of evaporation is properly taken into account.

4. Dependence on the Temperature of the Surrounding Air

Since the thermal inequilibrium is the main driving force of vibration, an experiment with various degree of inequilibrium may lead to deeper understanding. Motivated by this idea a new experiment was made. The temperature of the horizontal floor and the surrounding air were changed, by setting up the apparatus within a heater or a refrigerator, within the range from –87°C to 200°C, and behavior of a drop of liquid-nitrogen was observed by a video-camera. Size of the drop and vibrational frequency were measured by image processing from video images. Results of the measurement are summarized below.

Dependence of the frequency on the drop size is shown in Fig. 5, for several temperatures. It is remarkable that the data for the same mode lie nearly on the same curve, irrespective of the temperature difference of about 300 deg. The frequency did not depend on the ambient temperature but solely on the drop size. An only effect of different temperature was the long time-scale behavior, such as the life-time of a drop or behavior of mode transition.

Since the dynamics of vibration is governed mainly by the drop size (the inertia effect) and the surface tension (the restoring force), this result means that the surface tension of the drop was nearly constant independent on the ambient temperature T, i.e., the surface was kept at a common temperature irrespective of the ambient condition.

Figure 6 is a diagram showing processes of mode transition at various temperatures for drops of liquid-nitrogen. From this figure we can see that the process is the same in a wide temperature range $-80°C < T_0 < 150°C$. In a region

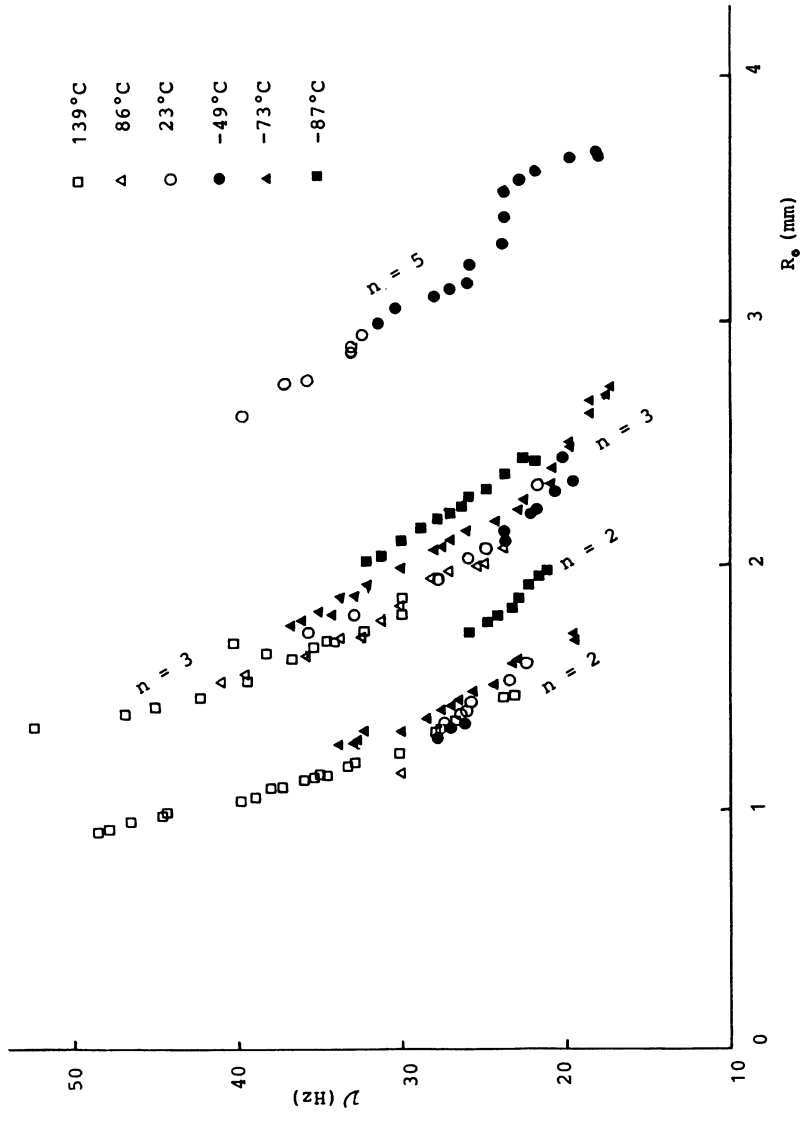

Fig. 5. Frequency of vibration plotted against the drop size.

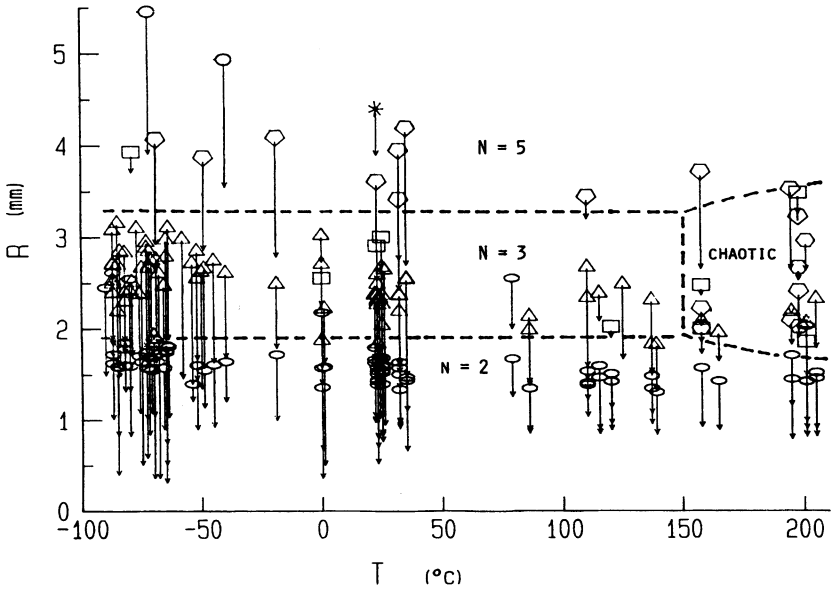

Fig. 6. Diagram of mode transition patterns.

with 150°C < T_0 and 1.5 mm < r_0 < 3 mm each mode had a regular pattern but was not selected uniquely so that the mode transition seemed to occur rather randomly. Probably, the process depended much on the initial condition. Moreover, 6-mode and 4-mode, which had never been observed in room temperature, appeared in this region. In that sense, this region can be called chaotic. It is not clear by what quantities this chaotic nature should be described.

On the other hand, for very low temperature drops did not show vibration. The critical temperature for occurrence of vibration is not yet obtained, though it seems to be around –100°C.

5. Discussion

In this article results of observation, linear analysis and model simulation are introduced. They gave us a lot of information on the phenomena, but the present status of the research is still not satisfactory and main problems are left unsolved, i.e., the mechanism of maintenance of vibration, that of mode selection and the physical state of evaporating surface. Important factors for excitation of polygonal vibration may be the variation of the surface tension coefficient, caused by the surface motion itself, and/or the thin vapor flow between the drop and the floor. Analyses of these factors are challenging and will be made by the authors in near

future.

In concluding this article, the significance of investigating the drop vibration is mentioned. Formation of macroscopic order in an inequilibrium systems is attracting interest of scientists with a name "dissipative structure" (Gransdorff and Prigogine, 1971). Are the patterns observed in evaporating drop the dissipative structures? The answer is "yes" in a sense that they occur in conditions out of thermal equilibrium. However, there is also an essential difference. In usual dissipative structure, degrees of inequilibrium are not so large that macroscopic fluid velocity is relatively small and the system shows in most cases only single mode for given values of parameters. The vibration of an overheated drop, on the contrary, is a phenomenon in a highly inequilibrium state. The drop shows not only a rapid motion, but also a successive transition to other modes.

Another characteristic feature is the change of system size. During the size change the drop made transitions to modes, which would be the most preferred ones for respective sizes. Such series of dynamical patterns can be looked upon as a pattern formation in the space-time framework. It may be an interesting concept to be investigated as a fundamental science.

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