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A Stochastic Model of Branching of Plant Trees

M. Agu and K. Yamanaka

*Faculty of Engineering, Ibaraki University**

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An evolution equation of stochastic spatial pattern of plant trees is proposed with taken into account of the two kinds of branches, apical branch and subsidiary one. The apical branch is assumed to have larger activity of bifurcation compared with that of subsidiary one. This phenomenon is observed in many branch-growth processes as apical dominance.

INTRODUCTION

The formation of tree structures has recently gathered attensions from the viewpoint of the pattern formation in non-equilibrium open systems. Especially, much interest has been forcussed on the fractal dimensions of the patterns of the tree structures found in crystal growth or dielectric breakdown (Mandelbrot:1982, Sawada et al:1982, Niemeyer et al:1984, Matsushita et al:1984). The structures of plant trees are also regarded as the patterns constructed in non-equilibrium open systems (Fisher et al:1979). In order to describe the growth of the tree structures, the bifurcation ability of branches known as apical dominance phenomenon and the stochastic property of bifurcations must be taken into account (Thornley:1977, Agu et al: 1985).

In the present report, an evolution equation of the spatial pattern of plant trees is proposed, in which the stochastic motions of the tips of the two kinds of branches, apical branch and subsidiary one are considered.

EVOLUTION EQUATION OF PLANT TREES

Let us assume that one and only one ancestor branch is located at position $x \in R^3$ and at the initial time t_0. The ancestor branch has the probability to bifurcate to give birth to two progeny branches, an apical branch and subsidiary one. The apical branch is assumed to bifurcate being subject to Poisson process with time-rate κ, giving birth to a new apical progeny

* Nakanarusawa 4-12-1, 316 Hitachi, Japan

branch and another subsidiary one. The subsidiary branch is also assumed to grow and become apical one following Poisson process with rate constant λ. The tip of the apical (or subsidiary) branch is assumed to develop to and fro randomly, following Markovian process with the transition probability T_+ (or T_-). Here, behaviors of different branches may depend on each other, i.e., either development or bifurcation of a branch may be affected by other branches. The stochastic processes considered above are summarized schematically in Fig.1.

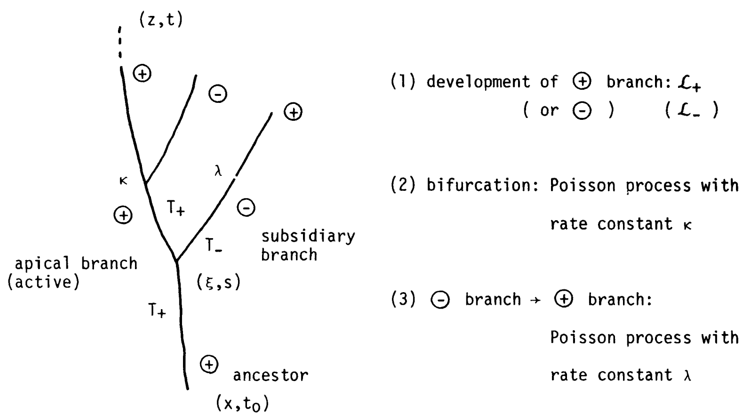

Fig.1. Branching process

Let $p_+(z,t|x,t_o)$ be a probability density such that an apical branch (⊕) originally situated at spatial point $x \in R^3$ and at the initial time t_o gives birth to a progeny branch at at spatial point z and at time t after some successive bifurcations. Similarly, $p_-(z,t|x,t_o)$ is the counterpart of $p_+(z,t|x,t_o)$ with respect to a subsidiary branch. The probability density $p_+(z,t|x,t_o)$ is divided into three independent parts: contribution of the apical branch born from the ancestor branch after the first bifurcation, that of the other subsidiary branch and direct contribution of the ancestor branch in the case free of bifurcations. Based on the method for the analysis of branching-diffusion processes (Iwasa et al:1984), the above idea is formulated as

$$\begin{aligned} p_+(z,t|x,t_o) = &\int d\xi \int_{t_o}^{t} ds\ e^{-\kappa(s-t_o)}\kappa T_+(\xi,s|x,t_o)p_+(z,t|\xi,s) \\ &+ \int d\xi \int_{t_o}^{t} ds\ T_+(\xi,s|x,t_o)e^{-\kappa(s-t_o)}\kappa p_-(z,t|\xi,s) \\ &+ \int d\xi\, e^{-\kappa(t-t_o)}T_+(\xi,t|x,t_o)p_+(z,t|\xi,t). \quad (1) \end{aligned}$$

Here, s denotes the first branching time and $T_+(\xi,s|x,t_o)$ the transition probability of the tip of the apical branch from the position x at the time t_o to position ξ at time s and κ the rate constant of the bifurcation following Poisson process. The first term on the right-hand side of Eq.1 represents the contribution of the apical branch born after the first bifurcation, the second term that of another subsidiary branch born after the first bifurcation, and the third term the contribution of the ancestor branch in the case without any bifurcations. The similar type equation for originally subsidiary branch situated at the position x at the time t_o is obtained as

$$p_-(z,t|x,t_o) = \int d\xi \int_{t_o}^{t} ds\, e^{-\lambda(s-t_o)} \lambda T_-(\xi,s|x,t_o) p_+(z,t|\xi,s) + \int d\xi\, e^{-\lambda(t-t_o)} T_-(\xi,t|x,t_o) p_-(z,t|\xi,t), \quad (2)$$

where $T_-(\xi,t|x,t_o)$ means the transition probability of subsidiary branch. Assuming the invariance on time-shift of the transition probabilities T_+, T_-, p_+, p_-, we have the evolution equations from Eqs.1 and 2

$$\frac{\partial}{\partial t} p_+ = L_+ p_+ + \kappa p_-, \quad (3)$$

$$\frac{\partial}{\partial t} p_- = (L_- - \lambda) p_- + \lambda p_+, \quad (4)$$

where L_+ or L_- is the evolution operator of T_+ or T_-, respectively:

$$\frac{\partial}{\partial t} T_+ = L_+ T_+, \quad (5)$$

$$\frac{\partial}{\partial t} T_- = L_- T_-. \quad (6)$$

An example of the computer simulation of two-dimensional spatial tree pattern is shown in Fig.2. Here, both apical and subsidiary branches are assumed to follow branching-diffusion processes with different diffusion constants. In the simulation, we made an additional assumption such that each subsidiary branch starts with tilt angle of 30° against the apical branch.

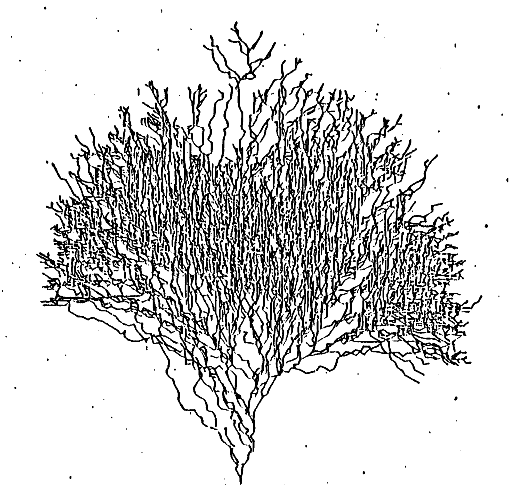

Fig.2. A computer simulation of tree structure

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