

New Models of Synergetics Topology and Their Reciprocal Space-filling Transformations

Yasushi Kajikawa

Design Science Institute

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All the unstable Archimedean polyhedral systems can be symmetrically transformed into Platonic polyhedral systems while still maintaining the same distances between the adjacent two vertices. Furthermore, these Platonic polyhedral systems can always ultimately be transformed into at least one of the three possible cases of fundamental omnitriangulated structural systems, viz. the tetrahedron, the octahedron and the icosahedron. Finally, the periodic relations inherent in these rational transformations can be reduced to "Structural Quanta". Next, the new dynamic topological frame models complex are constructed by complementary space-filler systems of Platonic and Archimedean polyhedra, which demonstrate the reciprocal space-filling transformations of four dimensional mobility.

In topology, Euler says in effect, all visual experiences can be resolved into three unique and irreducible aspects; vertices, faces and edges-or points, areas and edges. In terms of Synergetics Topology, they are called respectively joints, windows and struts (Fuller 1975a).

Topological analysis of synergetics explains all the multi-congruent --two, three, fourfold, fivefold-- topological aspects (Fuller 1979), by accounting for the primitive six vectors and four vertices of tetrahedron inventories of all Platonic and Archimedean polyhedral systems. Their respective primitive inventories of the topological aspect are always present at all phases of the rational convergence transformation of the Platonic and Archimedean symmetrical structural systems, which is demonstrated in the "Periodic Table of Synergetics Topology" and proves the quantum phenomena in terms of 3 Ground States of Synergetics Topology (Kajikawa 1984, 1985).

PERIODIC TABLE OF SYNERGETICS TOPOLOGY: STRUCTURAL QUANTA

The flexible joined regular and semi-regular unstable polyhedral systems can be folded into those with fewer regular plane windows, accomplishing the symmetrical collapsing of the same

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regular windows and ultimately into at least one of the multiple congruent tetrahedra, octahedra or icosahedra which are enclosed with the omnitriangulated windows. The only 3 possible omnitriangulated and omniequianguated structural systems are defined as 3 Ground States of Synergetics Topology.

Both the flexible joined regular and semi-regular vector-strut models lie with all the joints (vertices or points) in a containing sphere, the circumsphere. Each and every vector-length between the adjacent two joints are equal. There are only 3 possible types of flexible joints, because all of Platonic and Archimedean polyhedral systems can be sorted out by three, four or five vectors around each of their vertices; valency of the system. All models of polyhedral systems used in this research are made of these three types of flexible joints, windows and struts of the same length (Kajikawa & Sagara 1984b).

The following are full explanations of the data given in the periodic table.

Column 1) shows the polyhedral systems arranged in order of number of edges, from 1 to 18.

Column 2) contains the tetrahedron, the octahedron and the icosahedron constructed from equilateral triangles. They are the only 3 possible omnitriangulated structures in nature.

Column 3) contains the 15 unstable Plato-Archimedean polyhedral systems.

Column 4) gives the geometric name for each of the polyhedral systems.

Column 5) shows the number of edges for each polyhedral system.

Column 6) shows how all of the unstable regular and semi-regular polyhedral systems can be folded into those with fewer plane windows and ultimately into at least one of the 3 Ground States by accumulating vector-struts of the same number at each of their normal vector-struts. The numbers appearing in this column signify the number of multiple congruent polyhedral systems which will be formed by this continuous contracting process which involves the axial spinnability. The arrows, $x \rightarrow y$ mean that the continued folding of the figure x will result in y .

Column 7) show that when the numbers of vector-edges of each polyhedral system is divided by 6, the result will be one or more of the synergetics first four prime numbers, 1, 2, 3, 5 or multiples thereof (Fuller 1975a). There are either six vectors or none. Six vectors equal one minimum structural system. 6 edge-vectors = 1 structural quantum. (quantum means one of the small subdivisions of a quantized physical magnitude) The definable system of all Plato-Archimedean polyhedra is tetrahedrally coordinate in rational number increments of the tetrahedron.

Column 8) shows that in a quantum leap, the increase or decrease in the number of quanta will always appear in the order of the synergetics first four prime numbers, 1, 2, 3 or 5.

ROTATION OF FOUR AXES OF TRUNCATED CUBE: ARTICULATION OF EIGHT TRIANGULAR FACES WITH TWELVE EDGES

We can thread a nylon string through each of the 36 equal-length tubes twice to make a loop and fasten them together with three tubes joined at each of 24 corners to make the truncated cube, which proves to be structurally unstable. Tubular frame models of all Platonic and Archimedean polyhedra can be con-

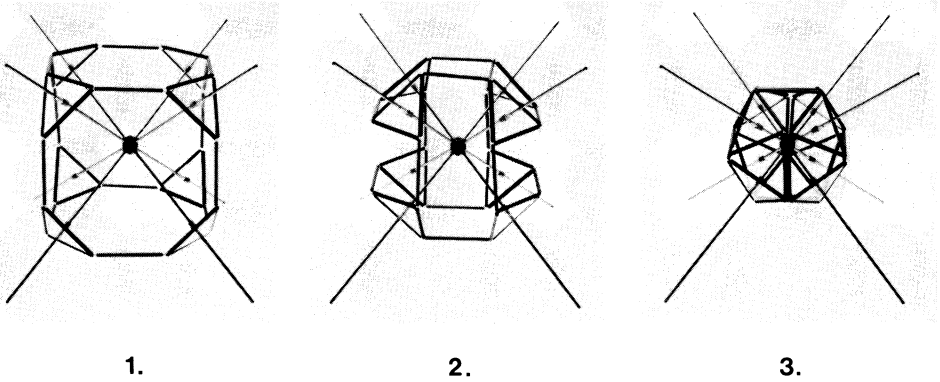


Fig. 1. Four Axes of Truncated Cube with Rotating Triangles

In the transformation from the truncated cube (1) to the icosahedron, each triangle rotates about these axes and approaches its center. As the truncated cube contracts, it transforms through the incomplete rhombicuboctahedral phase (2) and ends at the icosahedral phase (3).

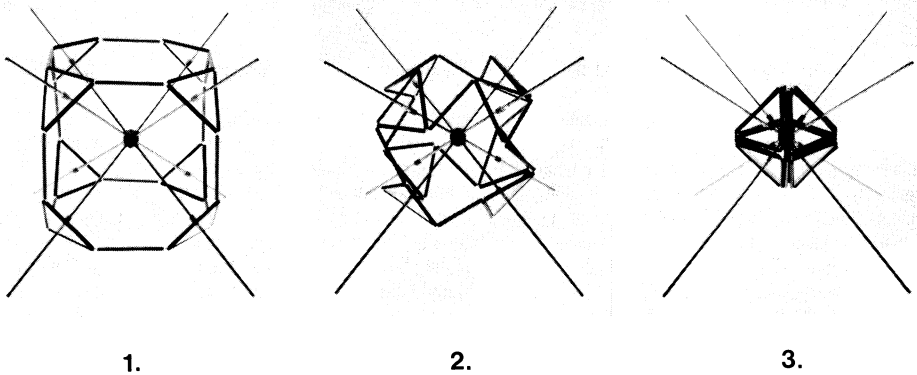


Fig. 2. Four Axes of Truncated Cube with Rotating Triangles

The term "truncated" in the names of most of Archimedean polyhedra refers to the new faces created by lopping off the vertices or the edges of the solid. However, in the dynamic frame models the lopping off neither increases nor decreases the total number of faces. By rotating the triangles we merely bring about either an expansion or a contraction of the model itself. By means of the rotation we are doing no more than opening or closing windows in the model. In the various contracting phases, each one of their vertices brings about a further circumspherical condition to accommodate the whole motion. As each triangle spins inwardly on four axes in the same direction, the truncated cube (1) transforms through the incomplete snub cube phase (2) and ends at the octahedral phase (3).

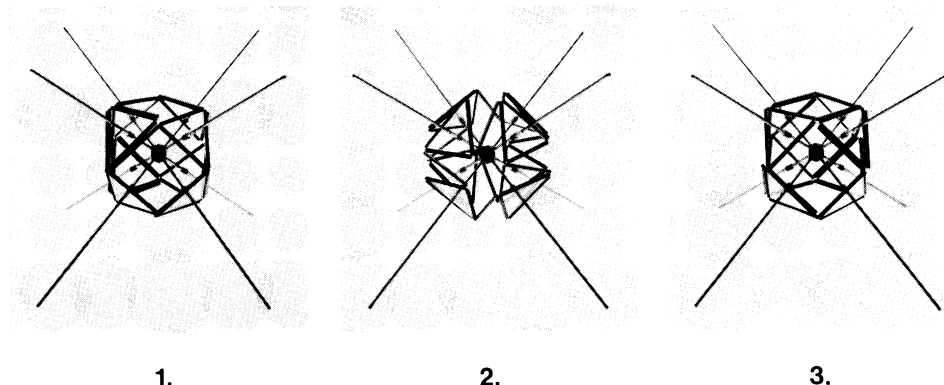


Fig. 3. Four Axes of Truncated Cube with Rotating Triangles

If four triangles which have tetrahedral configuration in the truncated cube spin inwardly toward the left or the right and other four triangles slide inwardly without spinning along the four axes, the whole system will contract until it becomes first cuboctahedral phase (1). When one triangle is turned in the opposite direction, the others also rotate automatically to interlock the whole system (2). Finally this link motion reverses the system to the potential alternate cuboctahedral phase again (3).

structed by using this loop-ligature technique (Kajikawa & Sagara 1984a).

We can have a truncated cube model made out of tubular frames colored in red, black and white with each of the eight transparent plastic triangles connected by four axes with a journal to slide on the shafts. Each shaft consists of a stainless steel rod which is perpendicular to two of the eight triangular faces. These models were invented in 1984-85. (a) The four axes of the truncated cube suggest a four-dimensional system. In the transformation from the truncated cube to the octahedron, each triangle rotates about these axes and approaches its center. The truncated cube lies with all vertices in a containing sphere; the circumsphere. In the contraction of the model, each one of its vertices brings about a further circumspherical condition to accommodate the whole motion. (b) If four red triangles of the model spin in left direction and four black triangles spin in right direction inwardly on four axes, the whole system will contract symmetrically until it becomes first the incomplete rhombicuboctahedral phase and finally the icosahedral phase. At this stage, the icosahedron has six sets of double white edges and a further twenty-four edges comprising eight sets of red and black triangles with eight spokes forming four axes running through the centers of the uncolored areas of the transparent plastic triangles. There is a direction of spin that throws a twist into the system --positive and negative. The right-handed icosahedron and the left-handed icosahedron are not the same. We can see that there are really two different icosahedra by means of coloring the vectors to

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identify them (Kajikawa & Sagara 1985) (See Fig. 1). (c) If all the triangles of the truncated cube model spin inwardly on four axes in the same direction, the whole system will also contract symmetrically until it becomes the incomplete right-handed or left-handed snub cube phase, and finally the octahedron phase. At this stage, the vector edges have tripled. There is also a direction of spin that throws a twist into the system --positive and negative. The right-handed octahedron and the left-handed octahedron are not the same (See Fig. 2). (d) If four red triangles of the truncated cube model spin inwardly in the left direction and four black triangles slide inwardly along the four axes without spinning, the whole system will contract symmetrically until it becomes the cuboctahedral phase, which has the four sets of double-edged red and white triangles and the four sets of single-edged black triangles. The eight triangles are always linked with twelve white edges between each of the two adjacent triangles and when one triangle is turned in the opposite direction, the others also rotate automatically to interlock the whole system. This link motion reverses the system to the potential alternate cuboctahedral condition again. But the new cuboctahedra have completely rearranged combinations of edges (See Fig. 3). If all the triangles of the cuboctahedra start to rotate about these axes, just as in the single cuboctahedra ("Jitterbug" in Fuller notation), the whole system will contract symmetrically until it becomes first the incomplete icosahedral phase, and finally the octahedral phase. At this stage, the vector edges have tripled, too.

SYMMETRICAL ROTATION OF SPOKE-AXES OF PLATONIC AND ARCHIMEDEAN POLYHEDRAL SYSTEMS

There are 14 new dynamic frame models of Synergetics Topology. (See "Periodic Table of Synergetics Topology" including the following axial spinnability of vertices or faces.)

- 1) Rotation of 1 Axis of Cube (See Fig. 4)
- 2) Rotation of 4 Spoke-Axes of Truncated Tetrahedron
- 3) Rotation of 1 or 4 Axes of Cuboctahedron (Fuller 1975b)
- 4) Rotation of 1, 4 Spoke-Axes or 4 Axes of Dodecahedron
- 5) Rotation of 1, 3 or 4 Axes of Truncated Octahedron
- 6) Rotation of 1 or 4 Axes of Truncated Cube
- 7) Rotation of 1, 3 or 4 Axes of Rhombicuboctahedron
- 8) Rotation of 1, 3 or 4 Axes of Snub Cube
- 9) Rotation of 1, 3 or 4 Axes of Icosidodecahedron
- 10) Rotation of 1, 3 or 4 Axes of Truncated Cuboctahedron
- 11) Rotation of 1 or 10 Axes of Truncated Dodecahedron
- 12) Rotation of 1,4 Spoke-Axes or 6 Axes of Truncated Icosahedron
- 13) Rotation of 1, 3, 4, 6 or 10 Axes of Rhombicosidodecahedron
- 14) Rotation of 1 or 4 Spoke-Axes of Snub Dodecahedron
- 15) Rotation of 1, 3, 4, 6 or 10 Axes of Truncated Icosidodecahedron


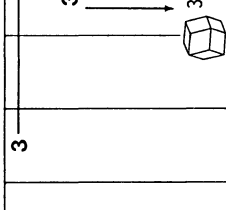
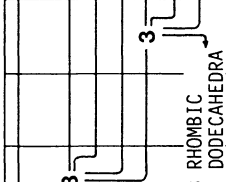
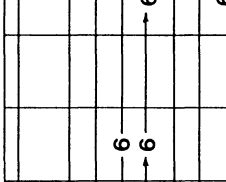
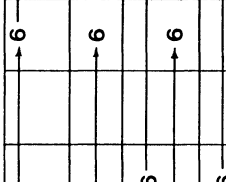
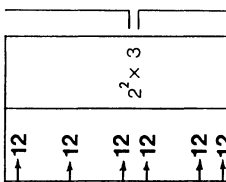






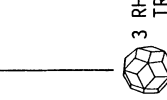
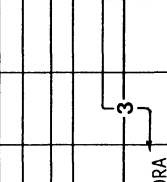
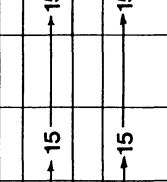
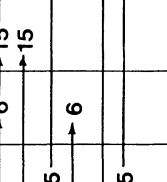
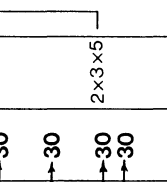
TRANSFORMATION OF TRUNCATED CUBE AND OCTAHEDRON AS SPACE-FILLING TRUNCATED CUBE

Because the truncated cube and the octahedron will fill space, it is possible to visualize a device for a space-filling "truncated cube" transformation. If we join many truncated cubes at their regular octagonal faces in a double space-filling

Periodic Table of Synergetics Topology: Structural Quanta

1	2	3	4	5	6						7	8			
	$3V - 6 = E$ $V = \text{Vertices}$ $E = \text{Edges}$	$3V - 6 < E$	Name of polyhedron	The number of vectors	The number of congruent polyhedra: Structural Quanta						Topological synergetics hierarchy	Quantum leap of first four prime numbers			
					2 Archimedean polyhedra	5 Doubling Platonic polyhedra			5 Platonic polyhedra			3 Ground States			
1			TETRAHEDRON	6									1	1	
2			OCTAHEDRON	12									1 → (2)	2	
3			HEXAHEDRON	12							1		→ 2	2	
4			TRUNCATED TETRAHEDRON	18									→ 2	3	
5			CUBOCTAHEDRON	24	1								→ 2	4	1
6			ICOSAHEDRON	30									→ 2	5	1
7			DODECAHEDRON	30							1		→ 2	5	1
8			TRUNCATED OCTAHEDRON	36									→ 2	6	1
9			TRUNCATED CUBE	36									→ 2	6	1
10			RHOMBI-CUBOCTAHEDRON	48	2								→ 2	8	2
11			SNUB CUBE	60									→ 2	8	2
12			ICOSI-DODECAHEDRON	60	1								→ 2	8	2
													→ 2	10	2
													→ 2	(5)	2
													→ 2	(10)	5
													→ 2	(10)	2
													→ 2	(10)	2

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13		TRUNCATED CUBOCTAHEDRON	72						2
14		TRUNCATED DODECAHEDRON	90						3
15		TRUNCATED ICOSAHEDRON	90						3
16		RHOMBICOSI- DODECAHEDRON	120						5
17		SNUB DODECAHEDRON	150						5
18		TRUNCATED ICOSI- DODECAHEDRON	180						5

Each of the 5 Doubling Platonic polyhedra binds two struts at each of the midpoints of its edges. Some of the transformations of Nos. 2, 6, 11, 12 and 17 require the detaching and rejoining of the vertexial connections. They are represented by means of (), i.e. irreversibility.

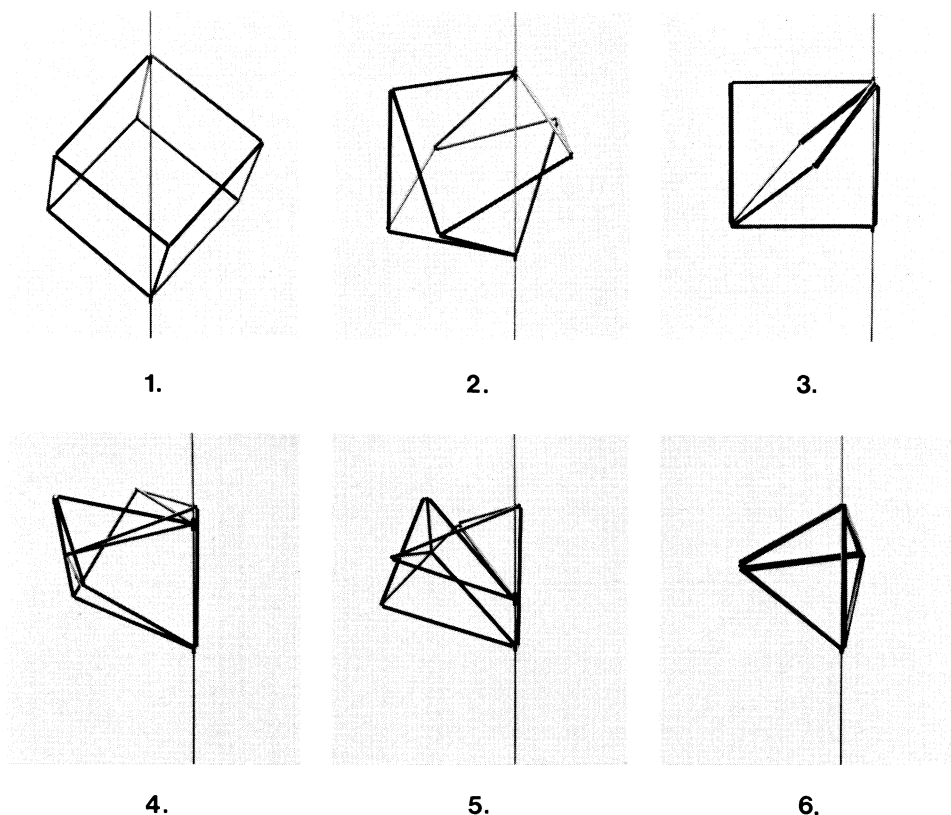


Fig. 4. Cube Folds into a Double Tetrahedra with Axial Spinnability of Two Poles

The axis transfixes the cube through the opposing north and south poles (1). When each pole rotates in the opposite direction,, the first phase is to bring together the two vertices which lie on the diagonal of the square (2). When, at the same time, this precessional movement occurs on the opposite side, the cube can be folded into an incomplete octahedron. In this case the two sets of double edges suggest polarization (3). Next, as the north pole moves on the axis of rotation toward the south pole, the second precessional movement occurs on the opposite side (4)(5). Finally, four pairs of opposing vertices come together to fold into two tetrahedra (6). The process of folding the cube is only dynamically symmetric with respect to the axis of rotation. The cube consists of a positive and a negative tetrahedron and is an indivisible unity. In other words, since the total number of edges and vertices of the cube is exactly twice that of the tetrahedron, we can refer to the cube as "two quanta" in our tetrahedral system. Ranking the models in terms of the number of edges, column 5 of the table shows how 18 types of Platonic and Archimedean polyhedra are all, without exception, composed of edges whose numbers are multiples of six. Multiplication occurs only through progressive fractionation of the original complex unity of the minimum structural system: the tetrahedron whose edges number precisely six.

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arrangement, the triangular faces form octahedral voids. After we put together a large omnidirectional complex of sets of four axes and eight transparent plastic triangles with twelve edges, we can interconnect the triangles of the octahedral voids from set to set with alternate sets of twelve edges (See Plate 1). As the truncated cubes contract towards each center, just as in the "single truncated cube", they transform through the rhombicuboctahedral phase and the cuboctahedral phase (See Plate 2, 3), which appears throughout the whole system as a triple space-filling arrangement of cuboctahedra, rhombicuboctahedra and cubes, and end at the octahedron phase (See Plate 4), which appears again as a double space-filling arrangement of truncated cubes and octahedra .

In other words, the original octahedra expand and ultimately become truncated cubes. Every truncated cube will become an octahedron and every octahedron will become a truncated cube on a large omnidirectional complex of sets of four axes. There is a complete change of the two figures. This oscillating motion makes an expanding and contracting system. In doing so, with a oscillating system and a pulsating circumspherical expansion-contraction going on everywhere locally, the whole system becomes an optically pulsating circumsphere.

Each exterior octahedron is a contracted truncated cube and is approximately one of the spaces between the circumspheres of the actual truncated cubes which overlap locally. Each octahedron thus becomes available as a potential alternate new circumsphere when the old circumspheres become spaces.

OTHER TYPES OF THE RECIPROCAL SPACE-FILLING TRANSFORMATIONS

"Rotation of 4 Axes of Vector Equilibrium" (vector equilibrium means cuboctahedron) was discovered by R.B.Fuller in 1944. In 1976 he further conceived and designed a limited edition of the metal sculpture "Jitterbug," demonstrating 4-D wave generation as the reciprocity of cuboctahedron and octahedra in a space-filling jitterbug. This was the first model of synergetics topology with the four-axis system of separate and combining transformations of local energy events (Fuller 1975c).

By using the other three types of the reciprocal space-filling transformations with the spoke-axis system (1: truncated tetrahedron + tetrahedron, 2: truncated cube + octahedron, 3: truncated octahedron + cube + truncated cuboctahedron) which I had the good fortune to discover and develop in 1984-85, we make a single energy action in the spinning system and a complete omnidirectional rotation occurs so that every one of the faces of the complementary space filler can shuttle back and forth. The distance between each of the two adjacent cores is always constant in both double and triple space-filling arrangements.

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Popko, E. (1968): Fig. No. 1. Geodesics. [Detroit. University of Detroit Press. 106] : 1

Plates 1, 2, 3, 4 are printed on Plate III at the opening of this volume. The following is an additional explanation to these plates.

In the space-filling "truncated cube" transformation, we find that if one force is applied to one triangle of one open truncated cube, the actual truncated cube closes to become an octahedron throughout the whole system and that each one of their vertices brings about a further spherical condition to accommodate the whole motion with a large omnidirectional complex of sets of four axes. Because in a space filling array there are equal numbers of truncated cubes and octahedra (Plate 1), the original truncated cubes contract and ultimately become octahedra (Plate 4). There is a complete change of the two figures. There is also a force distribution lag in the system. The distance between each of the two adjacent cores of the models is always constant in both double and triple space-filling arrangements.

13-4

Q: Do you know of any case where the beautiful transformations you describe are seen in a natural system? (N. Packard)

A: The development of synergetics topology did not commence with the study of the geometric structures of nature, seeking to understand their beautiful logic. From 1981 to 1983 inclusive I began to explore synergetics topology and developed it in pure mathematical principle. In other words, I did not copy nature's structural patterns.

But now I realize these developed structural principles as physical forms of nature. A study of the crystals at high pressure, the changes in atomic structures, will always show that they are based on the tetrahedron, the octahedron or the cube. The responses of a crystal structure to compression are shown as the phase transitions of these regular geometric clusters. If eight rhenium-centered octahedra are joined in a double spacefilling arrangement to get a rhenium oxide (ReO_3), the triangular faces of rhenium form a cuboctahedral void. In the symmetrical contraction of the void from the cuboctahedron to the octahedron at a pressure of about 5,000 atmospheres which changes from bond shortening to bond-angle bending, the eight rhenium-centered octahedra rotate about the four axes of the cuboctahedron. (Hazen, R. M. and Finger, L. W. : Crystals at high pressure. Scientific American., May, 1985: 87,89) It is also conceivable that the reduction in the volume of scheelite-type compounds (ABO_4) under increased pressure, which are composed of distorted cube and tetrahedron, is predominantly caused by the symmetrical contraction of the cube. (see Fig.4 -2,3.) Another case is the supersymmetric molecular cluster of 60 carbon atoms. (Kroto, H. W., Heath, J. R., Brien, S. C. O., Curl, R. F. and Smalley, R. E. 1985 : C(60) Buckminsterfullerene. Nature., 318 :162-163.) Concerning the vaporization process, Synergetics Topology suggests that how the tetrahedral system of carbon atoms can rearrange itself in order to get 12 pentagons and 20 hexagons, showing 60 vertices and 90 edges which construct a truncated icosahedron. (See Symmetrical Rotation of Spoke-Axes of Platonic and Archimedean Polyhedral System)

Many reappearances of the transformation of Synergetics Topology in scientists' discoveries at various levels of inquiry will confirm the mathematical coordinating system employed by nature.